Materials Theory

Arch 211 Statics and Strength of Materials

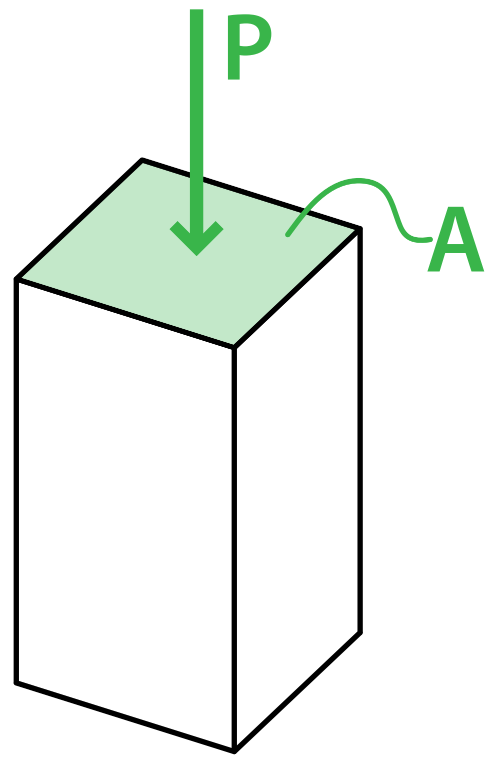
MATERIALS THEORY

Finding a way to describe the properties and behaviors of a material independent of a specific element or geometry.

STRESS

Quantity that describes the amount of force per unit area in a material sample. This is the primary quantity that governs whether a piece of material will fail.

$$f = \frac{P}{A}$$



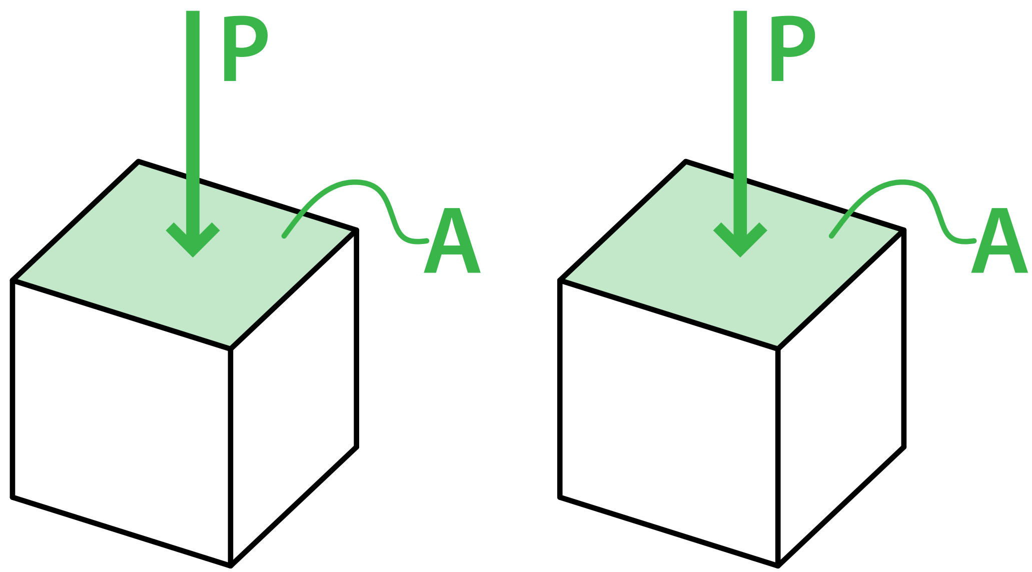
Stress is equal to the force acting on a material sample (P) divided by the area under stress. In the example to the right, it is the area perpendicular to P.

The units are N/mm^2, lb/in^2 or psi, and k/in^2 or ksi.

STRESS EXAMPLES

If we have a force (P) that causes a material sample to fail, it makes sense that to make 2 identical material samples fail, we need twice as much force.

$$f = \frac{P}{A}$$



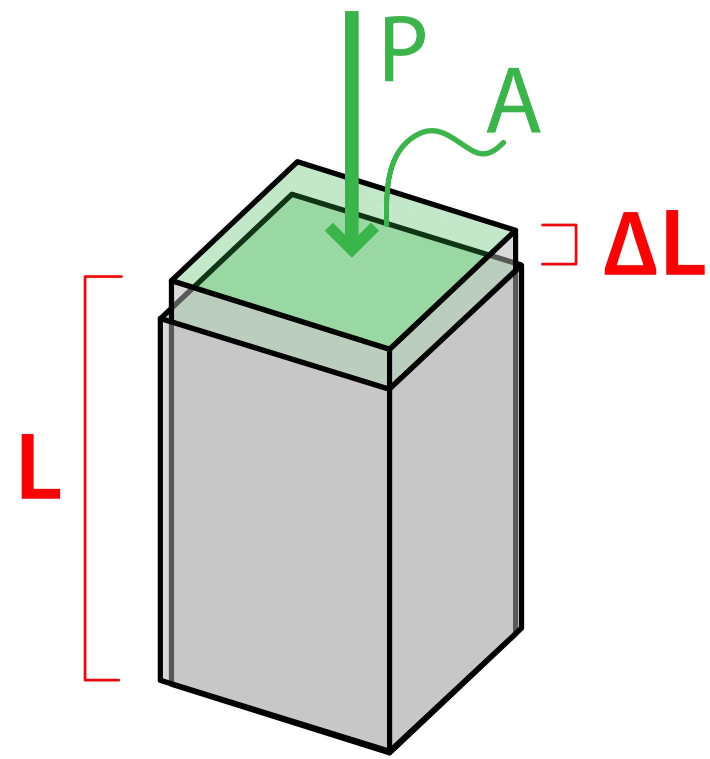
If one of those material samples has a cross-sectional area of 4 in^2, and a force of 1000 lbs compresses it, the material has a stress of :
f = 1000 lbs / 4 in^2 = 250 lb/in^2 (also 250 psi).

Stresses can occur in compression, tension, shear, bending, etc.

STRAIN

This is a quantity that describes the change in shape (deformation) of a material sample under stress.

$$\varepsilon = \frac{\Delta L}{L}$$



Strain is like a percentage change. It is the change in the length of a material sample divided by the original length of the material sample measured in the direction of the load (P).

Since L and ΔL are both measured in inches or mm, strain (ε) has units in/in or mm/mm, or can be described with no units at all.

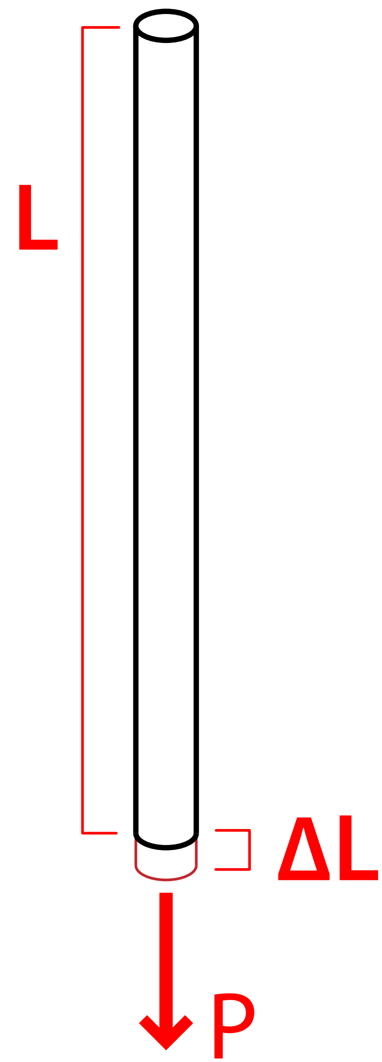
STRAIN EXAMPLE

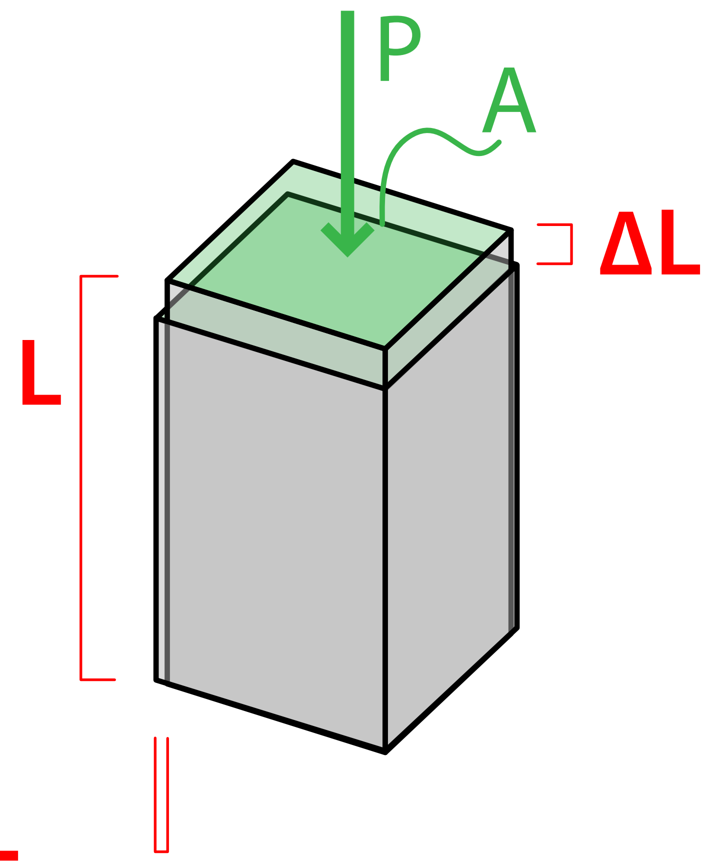
If a 1000″ long (83.3 ft) steel cable under tension extends by 1″, the strain is $\varepsilon = \Delta L / L = 1'' / 1000'' = 0.001$ (no units, or in/in).

$$\varepsilon = \frac{\Delta L}{L}$$

Stress and strain come together, and where one is present, the other will also be present.

Primarily, if a material sample is deforming (exhibiting strain), there are stresses in that material.

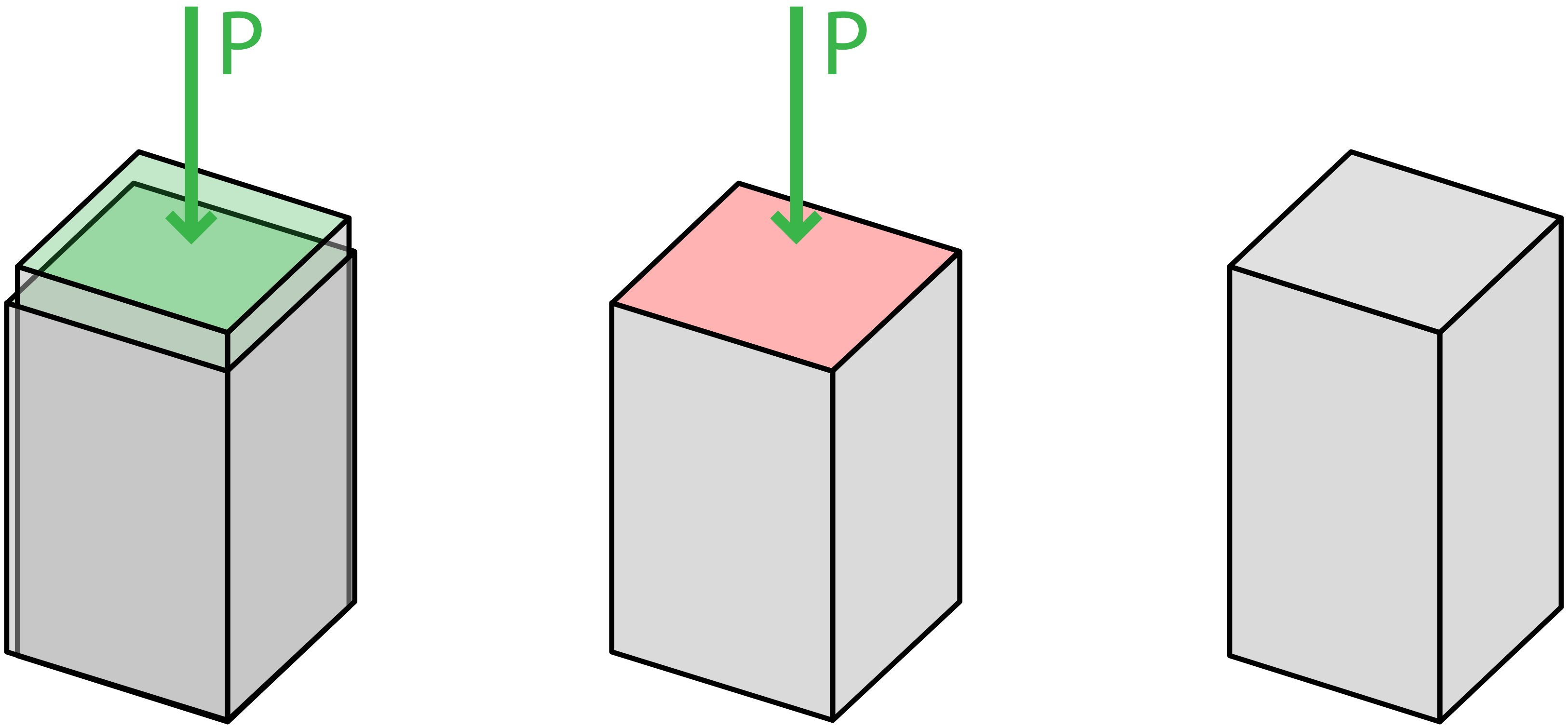




Materials sometimes expand when under compression (or contract under tension). This behavior is described by a quantity known as Poisson's ratio.

ELASTICITY

Describes the tendency of a material to return to its original shape once the stress in a material is removed.



Force is applied to the cross section of a material sample. This forms a compressive stress in the material.

The sample strains and deforms into a compressed state, shortening along the axis of the force.

If the force is removed, both the stress and strain disappear and the material returns to its original shape.

MODULUS OF ELASTICITY

Materials will begin to yield (deform beyond a point of no return) after a certain maximum stress (the yield stress). Below that stress is known as the "elastic range" of the material, in which the material exhibits elastic behavior.

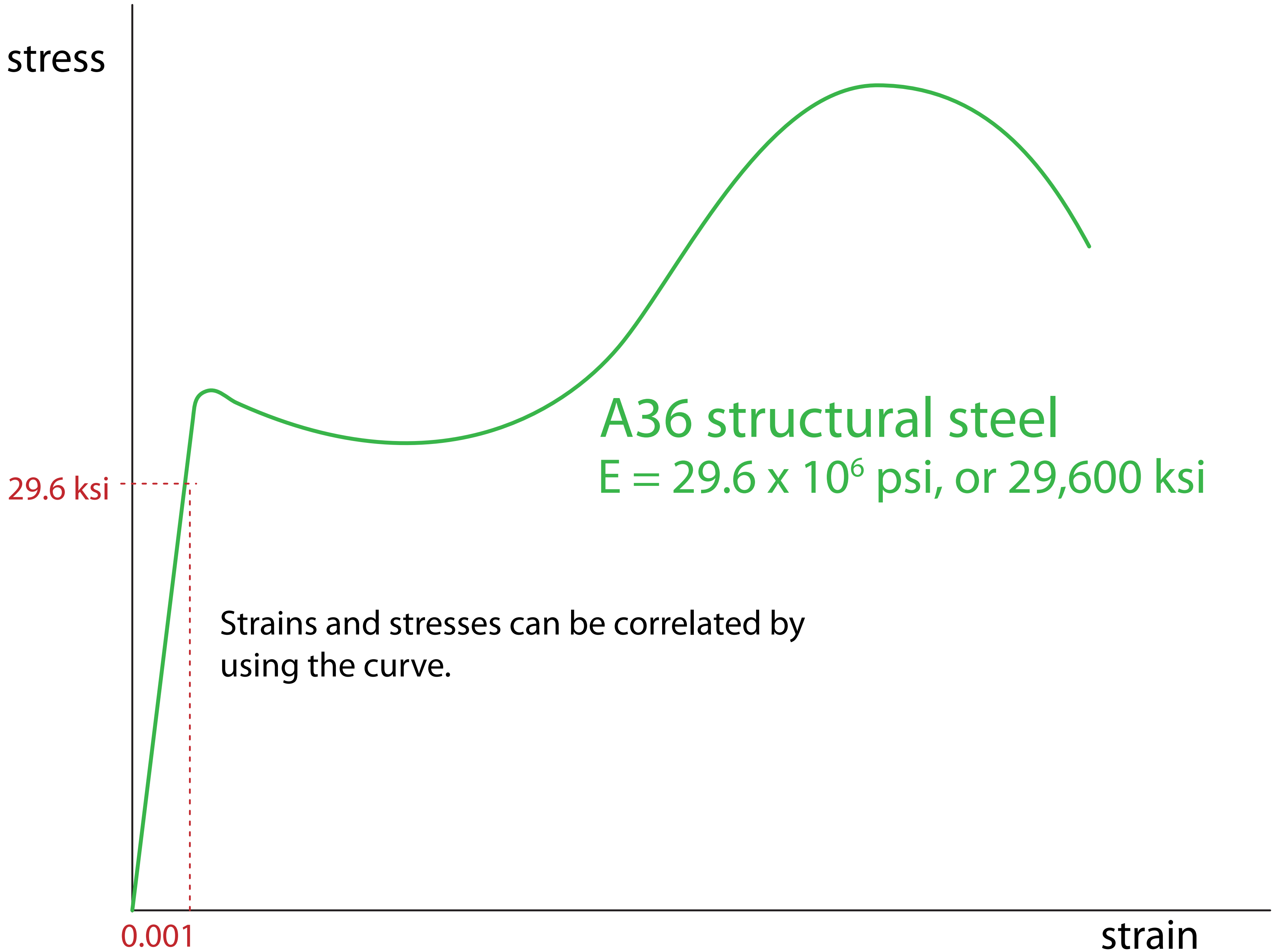
Most materials exhibit a special behavior according to what is called "Hooke's Law." This law states that within the "elastic range" of the material, the ratio of the stress divided by strain will be a constant (called the ***modulus of elasticity***).

$$E = \frac{f}{\varepsilon}$$

Definition of the ***modulus of elasticity***. The units are also psi or ksi. **This quantity serves as a defining quantity for a materials stiffness, since it relates loads to deformation.**

STRESS/STRAIN CURVE

A "fingerprint" for material behavior.



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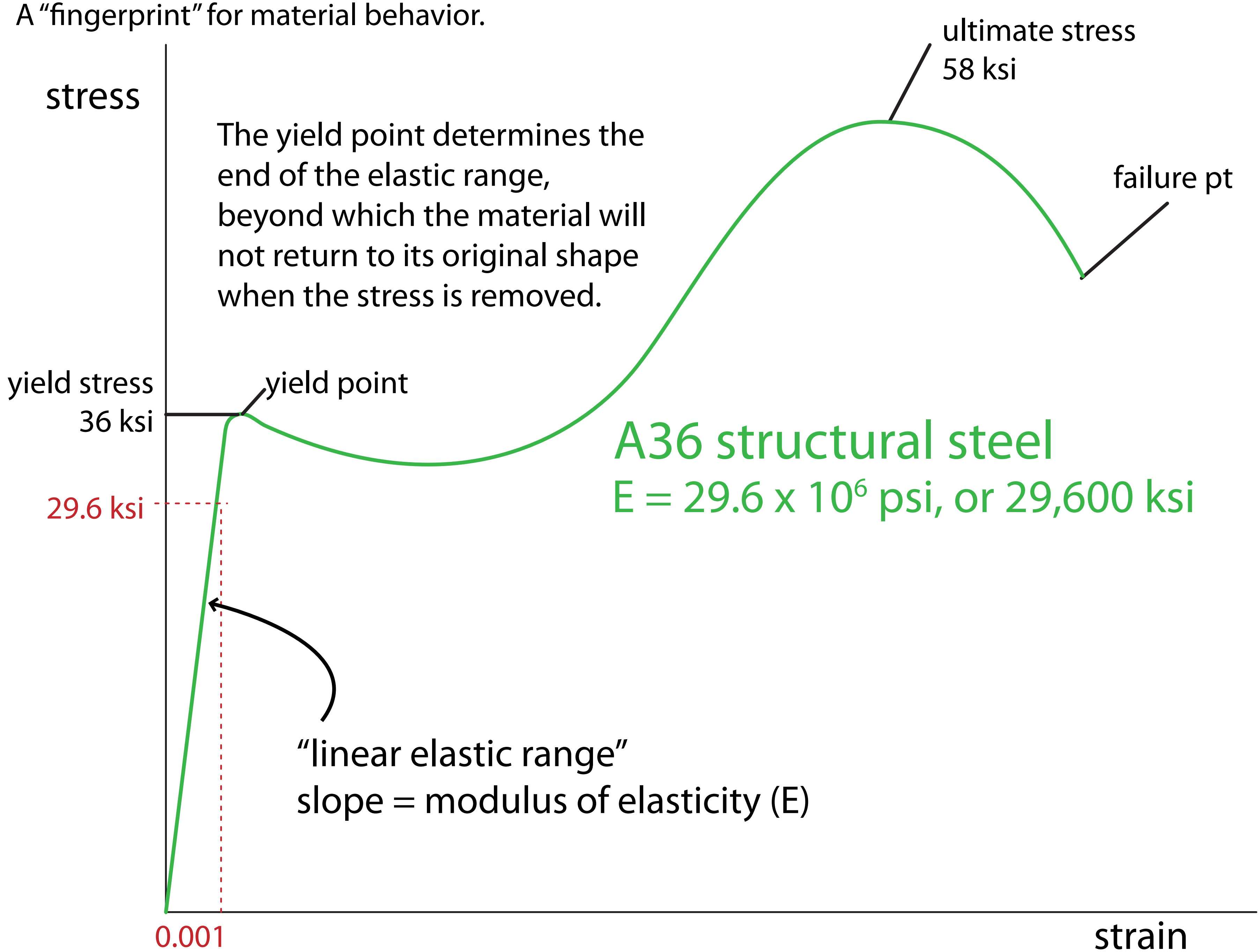
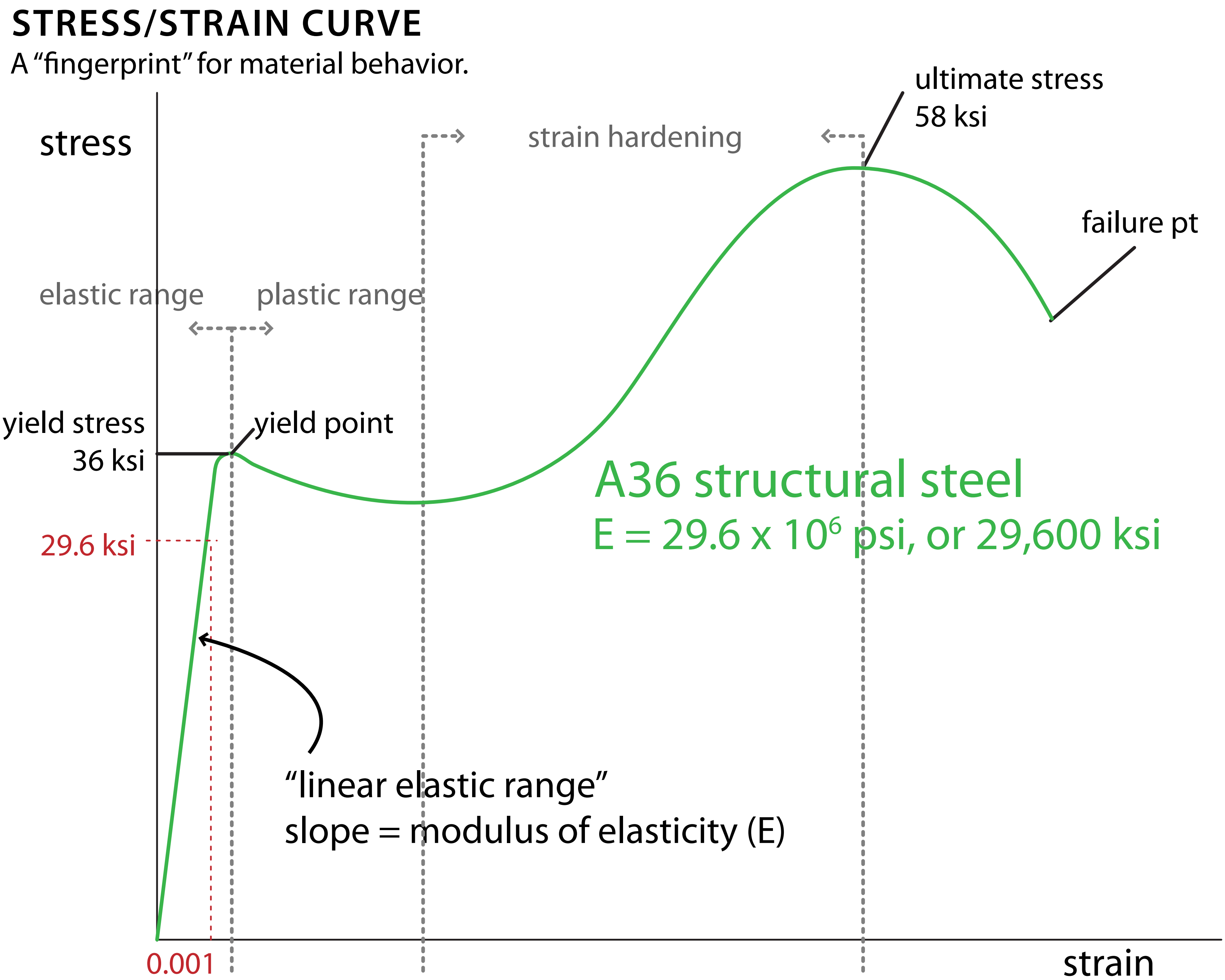



STRESS/STRAIN CURVE
A “fingerprint” for material behavior.
stress
ultimate stress
58 ksi
strain hardening
failure pt
elastic range
plastic range
yield stress
36 ksi
yield point
A36 structural steel
$E = 29.6 \times 10^6$ psi, or 29,600 ksi
29.6 ksi
“linear elastic range”
slope = modulus of elasticity (E)
0.001
strain

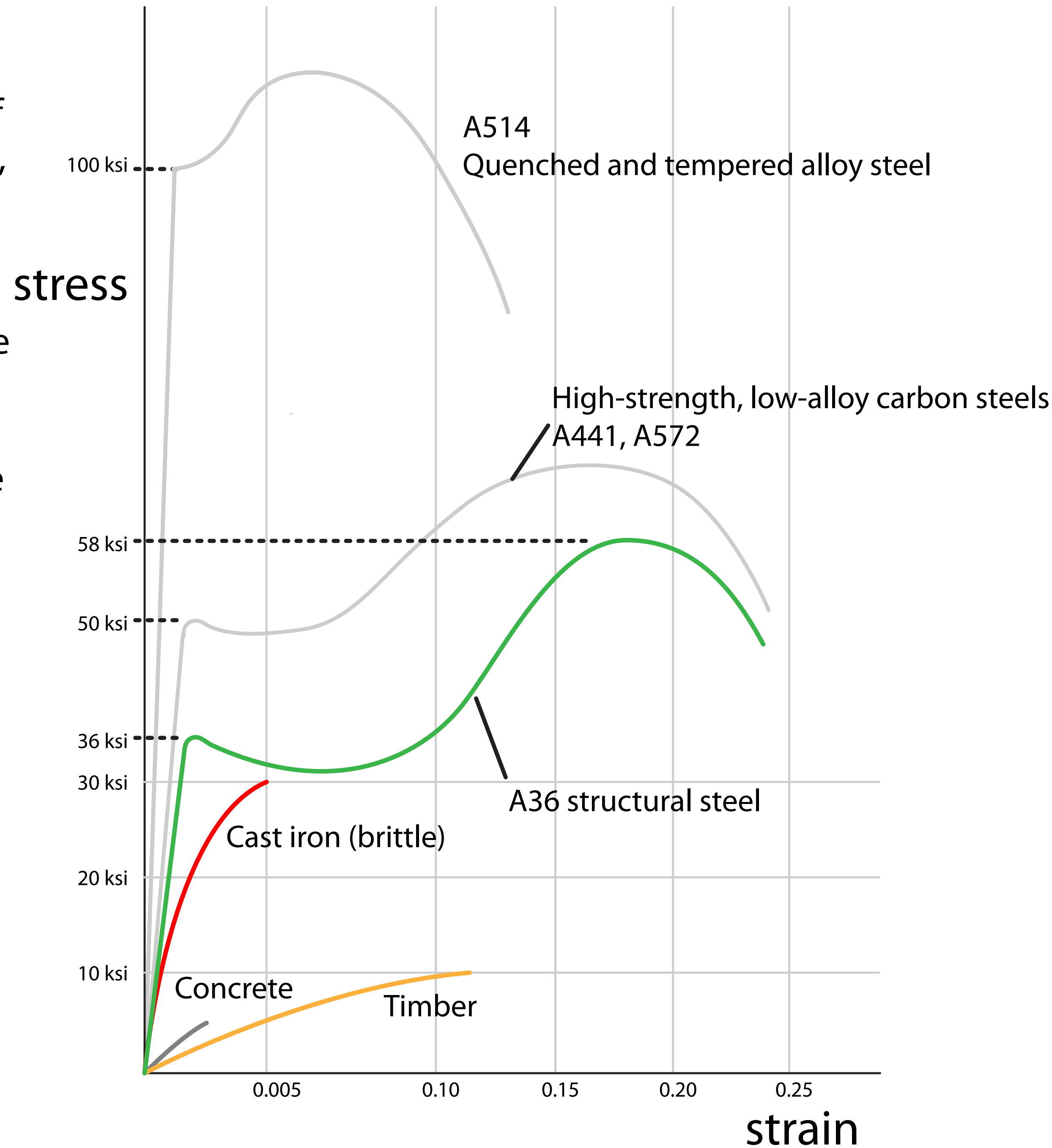
STRESS/STRAIN CURVE

Other materials and grades of steel are shown on this chart. From the shape of the curve and its extent along the axes, we can determine some important characteristics of the material.

How tall the curve goes determines the strength of the material.

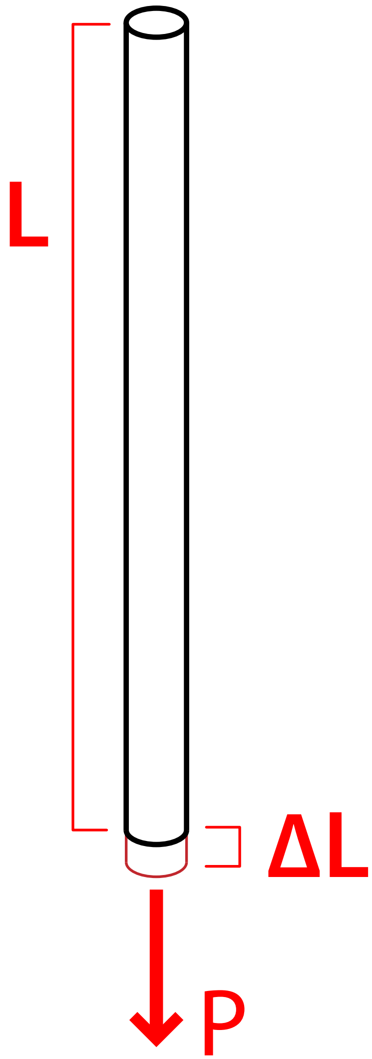
The slope of the first part of each curve determines the stiffness of the material. The steeper the curve, the stiffer the material, as it can experience high stresses with little strain.

Ductile materials tend to have longer curves along the strain axis. Materials that don't extend far on the strain axis tend to exhibit sudden, brittle failures.



SIZING TENSION MEMBERS

Tension members use the stress definition almost directly for their sizing. Using the definition of stress and the ***allowable stress*** of a given material (yield stress divided by a safety factor), we can solve for the cross-sectional area of the member that will be adequate to maintain elastic behavior.



Given f_{allow} = 22 ksi and P = 5,000 lbs, calculate the minimum cross sectional area for a steel cable.

$$f = \frac{P}{A} \quad \text{by definition.}$$

$$A = \frac{P}{f} = \frac{5{,}000 \text{ lbs}}{22 \text{ ksi}} = 0.227 \text{ in}^2$$

There are, of course, more technical details involved, but this gets us the first instance of how to size members using stress.