# LRFD Preliminary Beam Selection Procedure
## Moment and Deflection

## Calculate Design Loads

Determine dead, live, and other loads from the appropriate codes and manufacturer data.

For this example, assume:
dead load from floor slab = 50 psf
live load from occupancy = L = 100 psf (like a lobby or common area in a public building)
partition load = 20 psf
ceiling dead load = 10 psf

From the AISC Manual for Steel Construction, LRFD load calculations, choose the greatest magnitude of load after evaluating each expression. Each load factor responds to the uncertainty of the magnitude of the particular type of load. The greater the load factor, the greater the uncertainty.

D = dead load, L = live load from occupancy, $L_r$ = live roof load, S = snow load, R = rain or ice load (without ponding), E = earthquake load, W = wind load, U = design or ultimate load

1. U = 1.4D
2. U = 1.2D + 1.6L + 0.5 ($L_r$ or S or R)
3. U = 1.2D + 1.6 ($L_r$ or S or R) + (0.5L or 0.8W)
4. U = 1.2D + 1.6W + 0.5L + 0.5 ($L_r$ or S or R)
5. U = 1.2D ± 1.0E + 0.5L + 0.2S
6. U = 0.9D ± (1.6W or 1.0E)

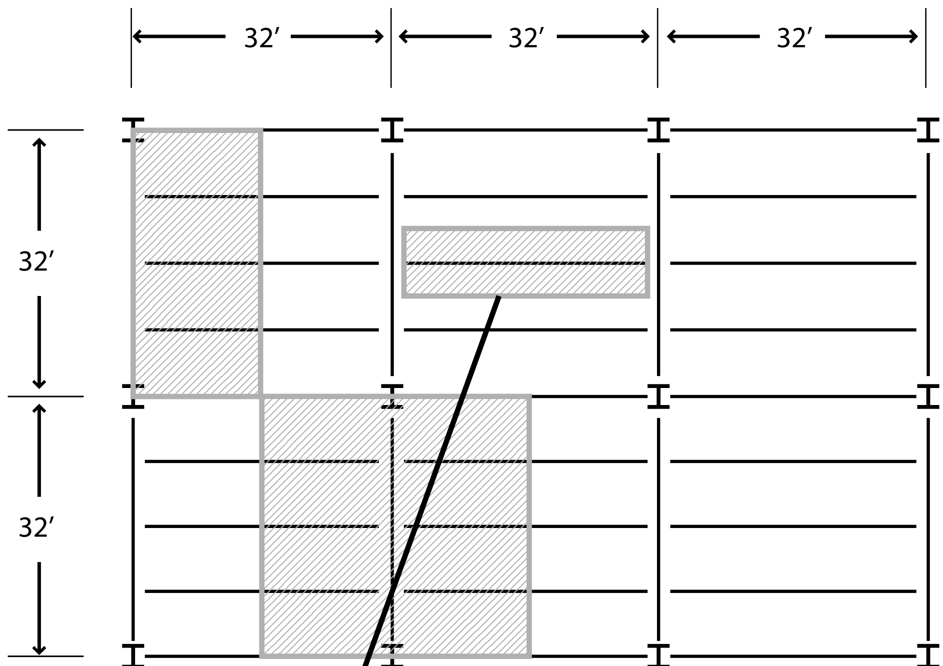
For our example:
D = 50 psf + 20 psf + 10 psf = 80 psf (beam self-weight is unknown)
L = 100 psf

Without the beam self-weight:
1. U = 1.4(80 psf) = 128 psf
2. U = 1.2(80 psf) + 1.6(100 psf) = 256 psf
3. U = 1.2(80 psf) + 0.5(100 psf) = 146 psf
4. U = 1.2(80 psf) + 0.5(100 psf) = 146 psf
5. U = 1.2(80 psf) + 0.5(100 psf) = 146 psf
6. U = 0.9(80 psf) = 72 psf

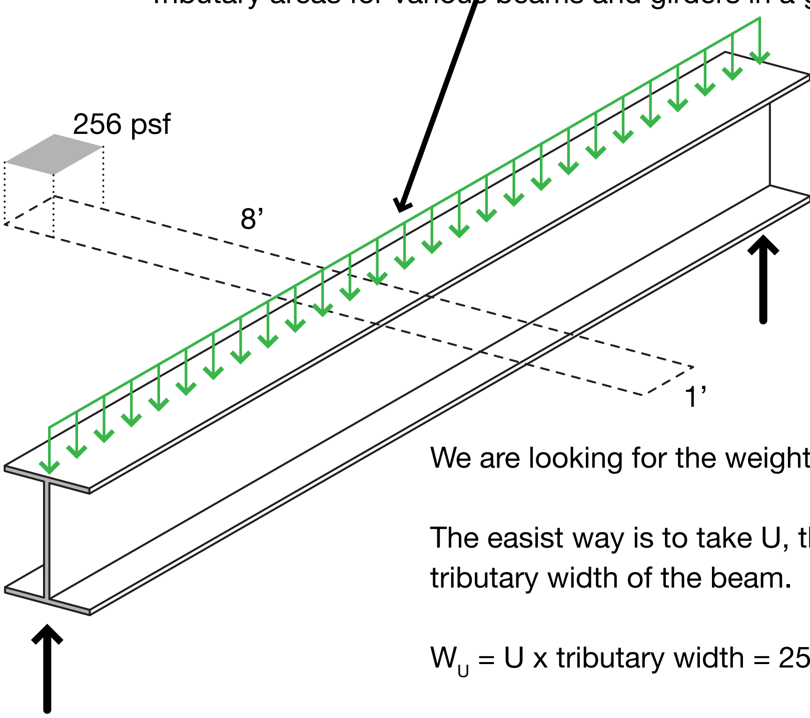
Since the problem doesn't specify wind, seismic, or roof loads, this seems trivial. Regardless, however, we have a maximum value that we can use as a design load from equation #2, 256 psf.

## Tributary Area and Uniform Distributed Loading

To place this value in the context of beam sizing, we need to look at the beam layout to determine the tributary area of the beam we are attempting to size. For a typical interior beam, spaced 8' on center, the tributary area is the length of the beam (32') by the width of the midlines between beam centerlines (8'), just the beam spacing itself.




Tributary areas for various beams and girders in a generic steel grid.



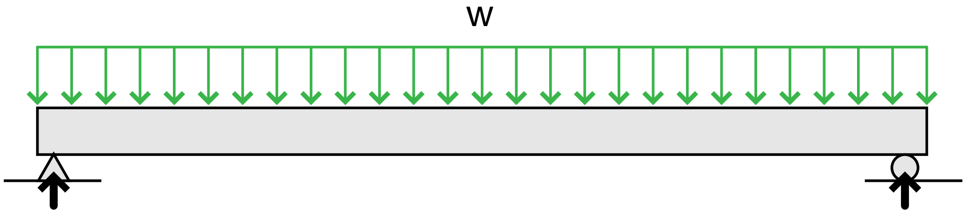

We are looking for the weight per linear foot on the beam, in k/ft.

The easist way is to take U, the design load, and multiply it by the tributary width of the beam.

$W_U$ = U x tributary width = 256 lb/ft$^2$ * 8 ft = 2048 lb/ft or 2.048 k/ft

## Plastic Moment

Given the uniformly distributed load, $W_U$ or w, we can calculate the internal bending moment present in the beam due to loading. Just the equation from the beam charts will help here.




$$M_U = \frac{wL^2}{8} = \frac{(2.048\text{ k/ft})(32\text{ ft})^2}{8} = 262.144\text{ k}\cdot\text{ft}$$

In order to size this beam, the following expression must be true.

$M_U < \phi_b M_{px}$ where

$\phi_b$ is the *resistance factor*, equal to 0.9 (accounting for uncertainty in the strength of the beam), and

$M_{px}$ is the maximum plastic moment resisting capacity of the beam, calculated by the following flexure formula:

$M_{px} = F_y Z_x$ where $F_y$ is the yield stress for the steel (here, 50 ksi), and $Z_x$ is the *plastic section modulus* for the section, which can be obtained from the steel dimensions chart.

If we combine the above expressions, we can solve for a minimum acceptable value for Zx, then select the appropriate cross section from the steel manual's chart. Normally, we look for the lightest section.

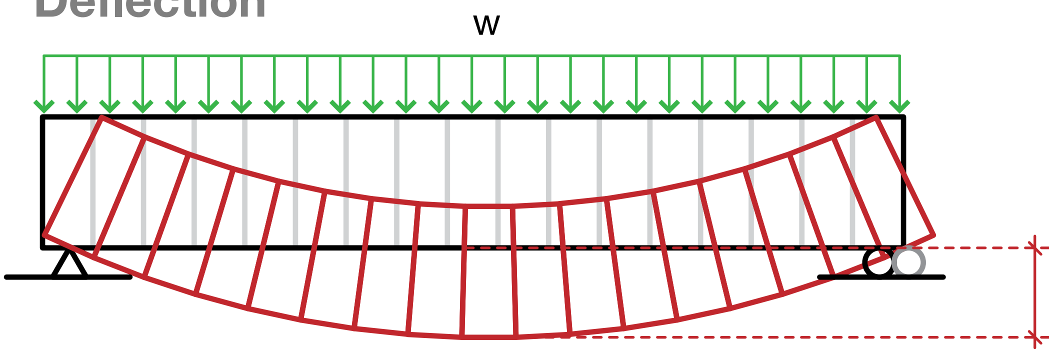
$$\phi_b M_{px} > M_U$$

$$\phi_b F_y Z_x > M_U$$

$$Z_x > \frac{M_U}{\phi_b F_y} = \frac{262.144\text{ k}\cdot\text{ft (12 in/ft)}}{0.9 \cdot 50\text{ ksi}} = 69.9\text{ in}^3$$

The lightest section just larger than 69.9 $\text{in}^3$ for the $Z_x$ value is a **W18x40**, which has a $Z_x$ value of 78.4 $\text{in}^3$. Now that we know the beam self-weight, 40 lb/ft, we can check the answer by doing the same calculation but adding this value to U (times the dead load factor of 1.2). This yields a minimum $Z_x$ of 71.5 $\text{in}^3$, still under our selected value of 78.4 $\text{in}^3$.

## Deflection




$$\Delta = \frac{5wL^4}{384EI}$$

Given the uniformly distributed load, $W_U$ or w, we can calculate the deflection of the middle of the beam given the formula above.

w = value of the uniformly distributed load (summed *without* the load factors)
L = span of the beam
E = modulus of elasticity for steel = 29.6 x $10^6$ psi (or 29,600 ksi) for all grades of steel
I = moment of inertia for the selected section, obtainable from the AISC sections dimensions chart

Deflections fall into the serviceability category, which means that we use the raw service loads (D, L, W, etc.) instead of the factored loads. These calculations are identical for ASD and LRFD methods.

Building code specifies for steel beams a maximum deflection of L/360. We simply have to calculate this allowed deflection and compare it to the actual predicted deflection above.

$$\Delta_{allow} = \frac{L}{360} = \frac{32 \text{ ft x } 12 \text{ in/ft}}{360} = \frac{384''}{360} = 1.067 \text{ in}$$

*Being careful to keep all units as pounds and inches:*

w = D + L = ( 80 psf + 100 psf ) x tributary width = 180 psf x 8 ft = 1440 lb/ft = 120 lb/in
From the steel chart, we can find $I_x$ = 612 $in^4$ for a W18x40.

$$\Delta_{actual} = \frac{5wL^4}{384EI} = \frac{5 \cdot 120 \text{ lb/in} \cdot (384 \text{ in})^4}{384\,(29.6 \text{ x } 10^6 \text{ psi})\,(612 \text{ in}^4)} = 1.875 \text{ in}$$

Our actual deformation has exceeded the allowable. This means we must select a new beam based on this new information. Working backwards, we can solve for a minimum value of $I_x$ of 1076 $in^4$.

$$\Delta_{allow} > \Delta_{actual} = \frac{5wL^4}{384EI_x} \quad \Rightarrow \quad I_x > \frac{5wL^4}{384E\Delta_{allow}} = 1{,}076 \text{ in}^4$$

The next larger section that will work is a W18x71, with an $I_x$ of 1170 $in^4$. A lighter (and thus more preferable) selection, proceeding up the chart, would be a **W21x55**, with an $I_x$ of 1140 $in^4$. Note that these both still meet the $Z_x$ plastic section modulus requirement.