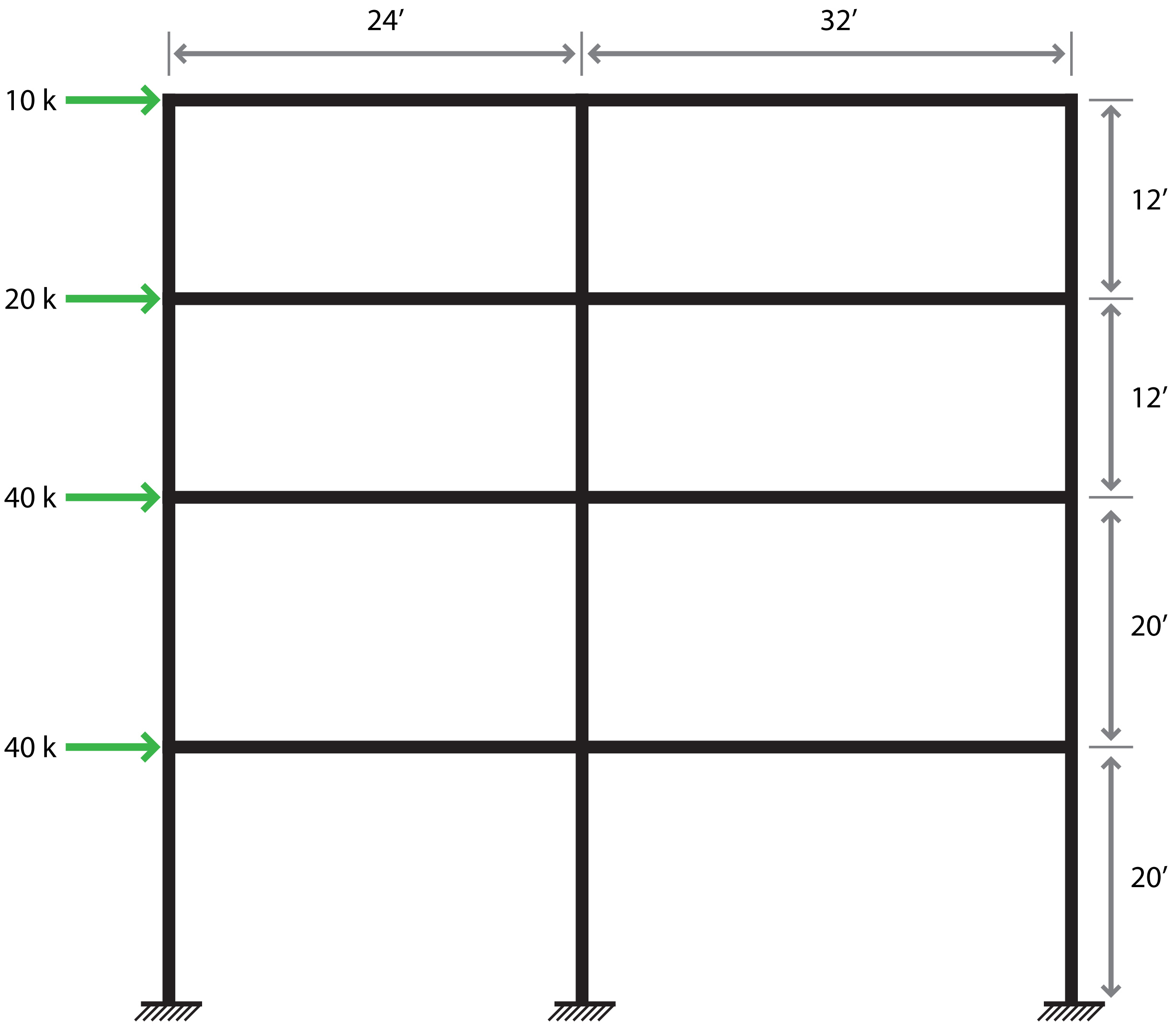
Portal Frame Calculations
Lateral Loads

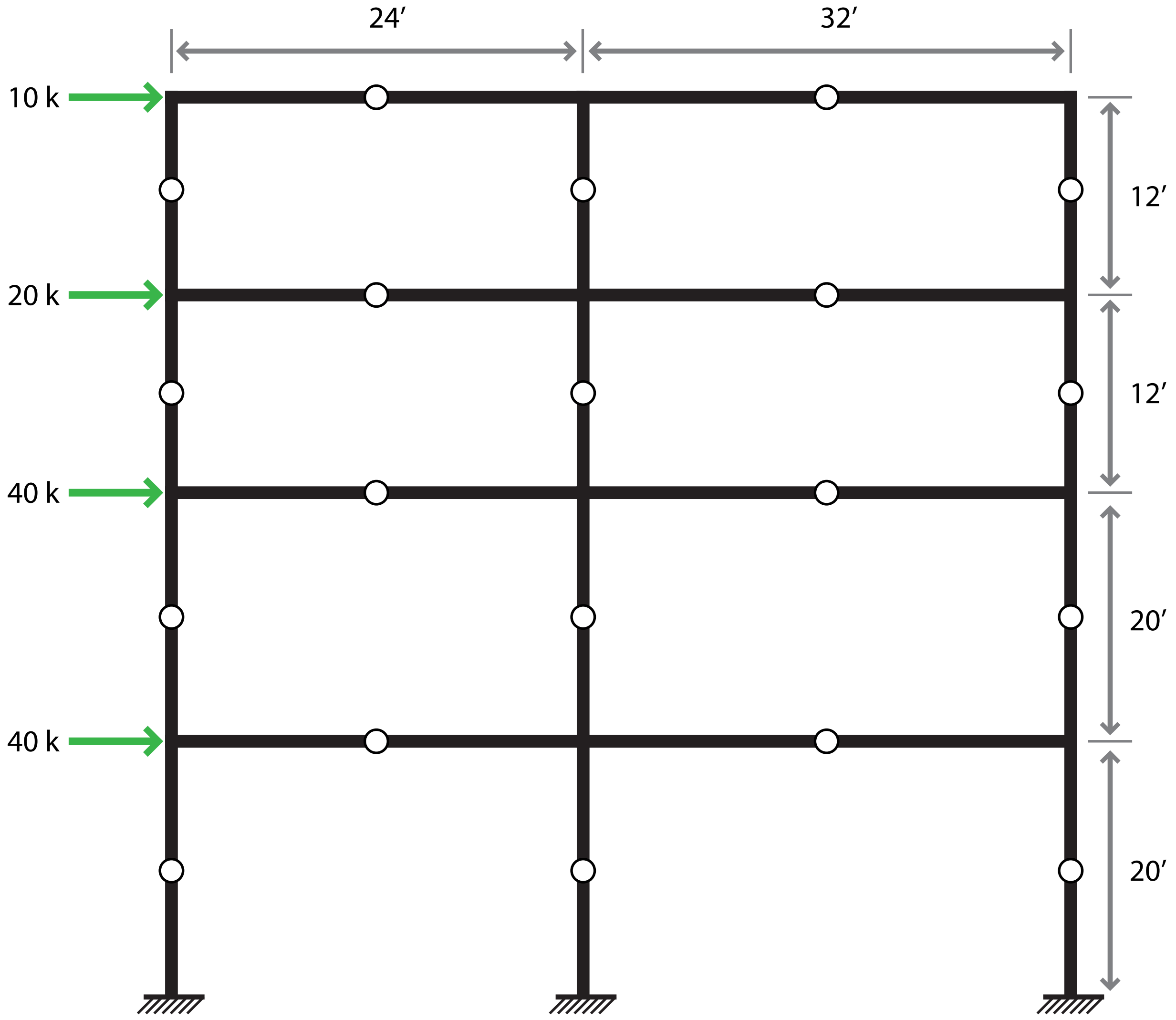
Consider the following multi-story frame:



The portal method makes several assumptions about the internal forces of the columns and beams in a rigid frame:

1) Inflection points for beams and columns are in the centers of the spans/lengths.
2) Internal columns take exactly twice the shear of external columns.

Note inflection points for the frame:



Calculate shears first, from the top of the structure down. Cut at the inflection points first.
Note the unknown shears at each column (V for exterior columns, 2·V for interior columns).

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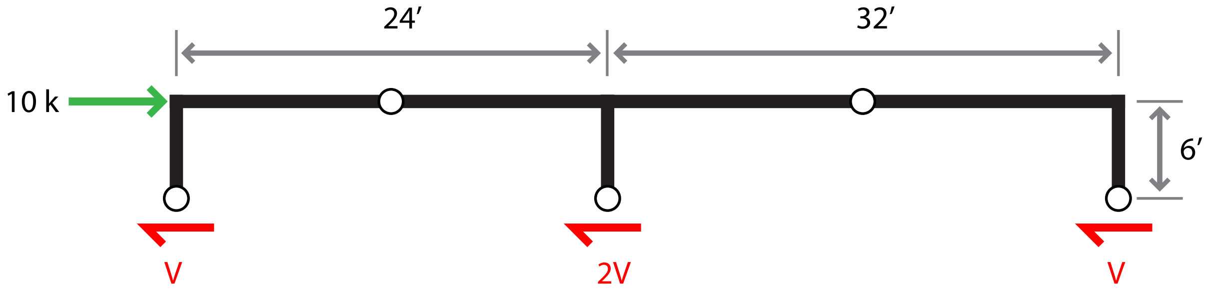
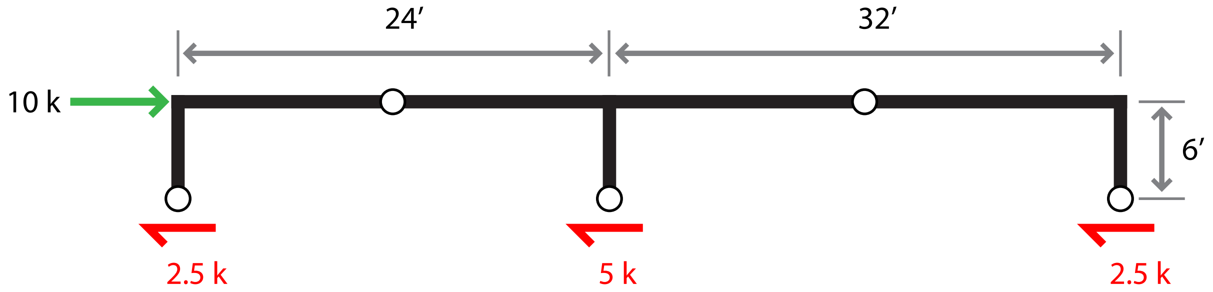


Due to statics, $\Sigma F = 0$, so $10k = V + 2V + V = 4V$. Thus $V = 2.5k$

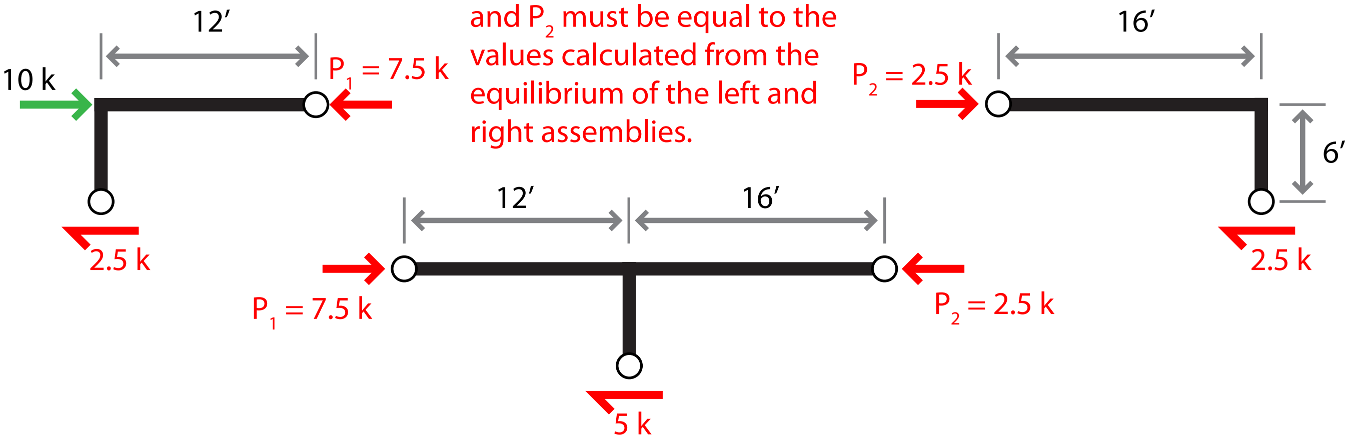


Break this partial frame into three assemblies, along the beam inflection points. Solve for the compression in the beam by resolving the unknown forces at the inflection points.

For the left assembly, solve for P_1.
$\Sigma F = 0 = 10\text{ k} - 2.5\text{ k} - P_1$.
$P_1 = 7.5\text{ k}$

For the right assembly, solve for P_2.
$\Sigma F = 0 = P_1 - 2.5\text{ k}$
$P_2 = 2.5\text{ k}$

For the middle assembly, P_1 and P_2 must be equal to the values calculated from the equilibrium of the left and right assemblies.



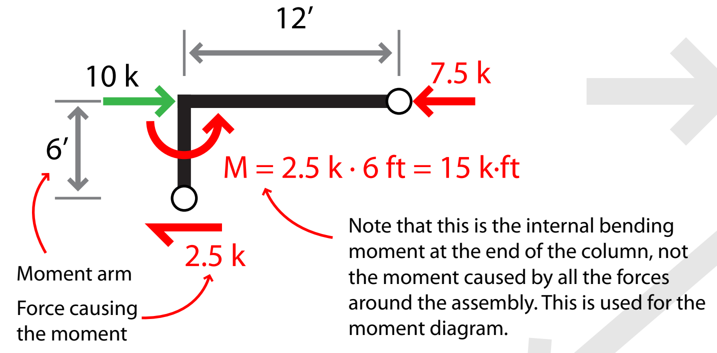
Use the middle assembly to check. The $\Sigma F = 0$ should hold given the shear in the column. You can also work left to right (left, middle, then right). The point is that the equilibrium of one assembly affects the equilibrium of the next.

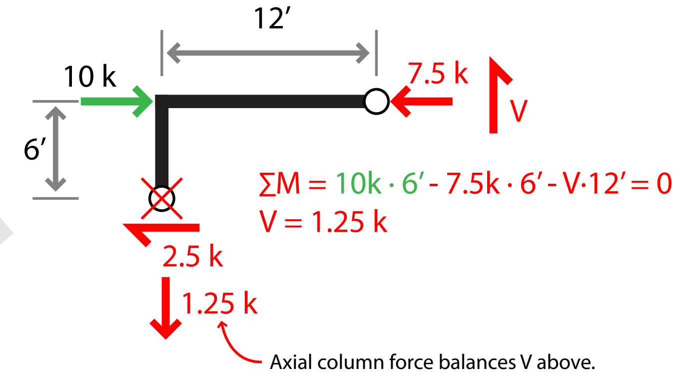
Calculate [internal bending] moments and vertical shears for the columns and beams. Moments come from the column shears and the half column height (6′). Vertical shears are calculated by balancing the moment across the corner, dividing by the vertical shear's moment arm (12′ on the left). All moments are measured from the given joint.

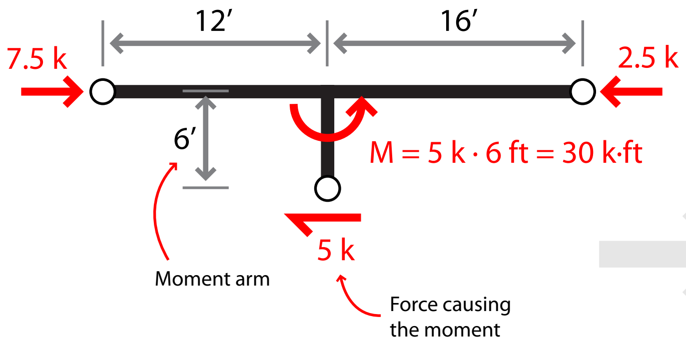
CALCULATE MOMENTS…

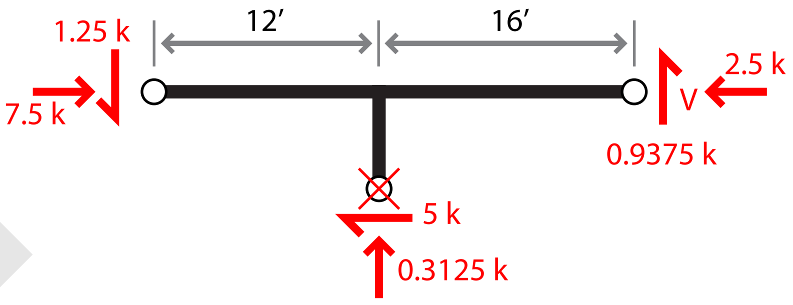
Moment arrows are drawn as the internal bending moment that *opposes* the corresponding shear in the column.

…CALCULATE RESULTING SHEARS AND AXIAL FORCES









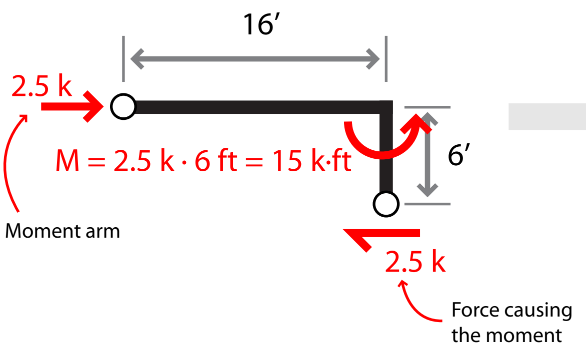
Set up the equilibrium to calculate V:

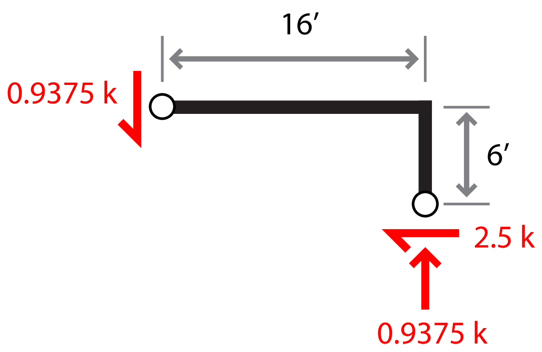
$\Sigma M = (7.5\text{ k})(6') - (2.5\text{ k})(6') - (1.25\text{ k})(12') - 16V = 0$

$= 15\text{ k·ft} - 16V = 0$

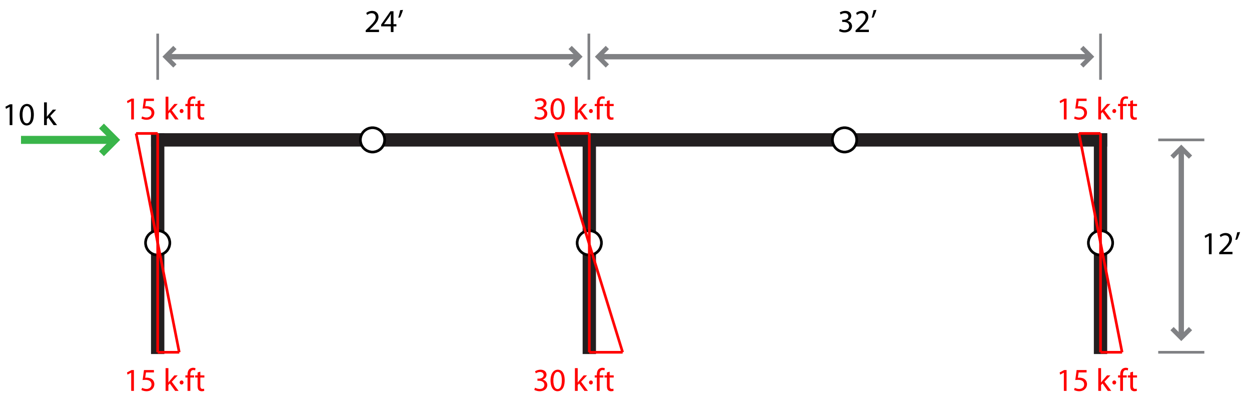
$V = 0.9375\text{ k}$

After we have V, we can calculate the axial load in the column.

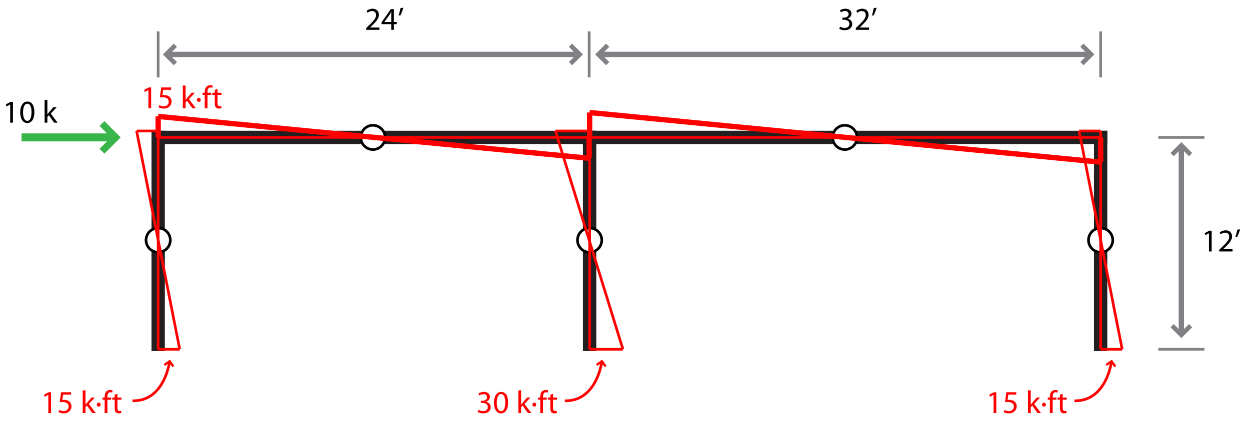




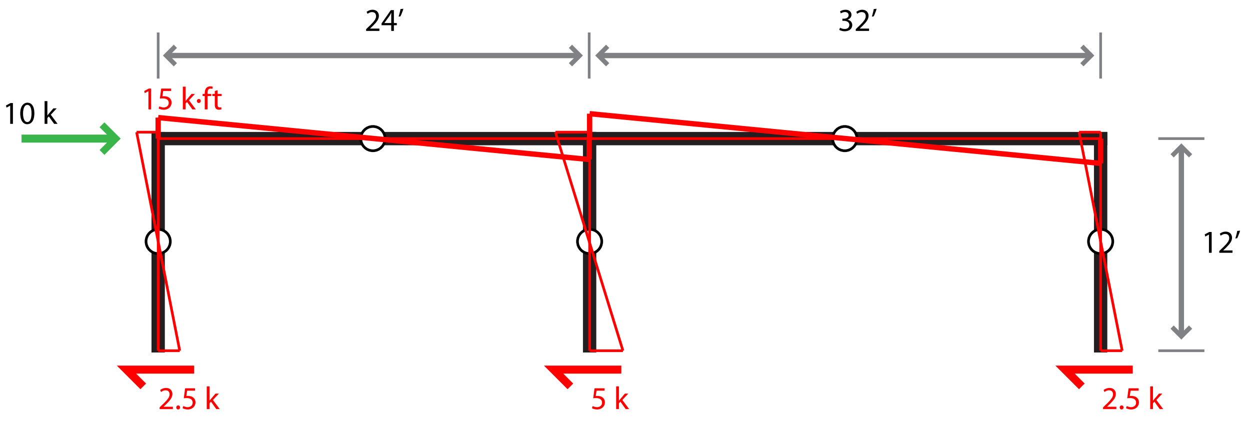
These moments are transferred down each column. Since the inflection points are assumed to be at the midheights of the columns, the moment at the base of each column (that is, where it meets the next beam) is equal to the moment at the top (opposite direction of course).



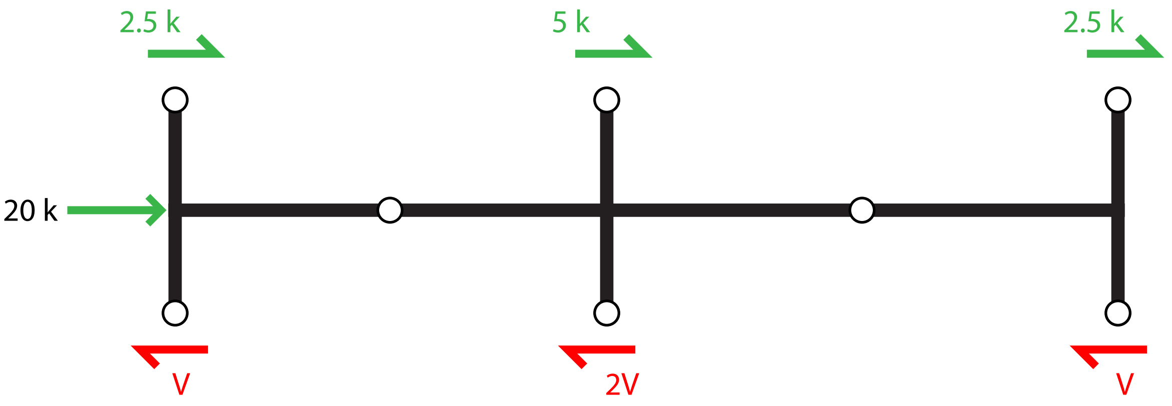
Given these moments, we can also calculate moments in the beams. The values of the moment on all sides of a given rigid connection must add to 0.



Before proceeding to the next floor, make a note of the shears again calculated at the start. The have the same values even though we are cutting at the bottom of the columns instead of at the midheights:



For the next floor, we go through the same procedure as the top floor. The only difference is the presence of the bottom half of the top floor columns. This tends to add more shear and moment to the equations, so the resulting magnitudes are a result of the lateral load shown and the moments and shears in the floor above. First, divide the next set of beams and columns along the inflection points. Note the shears from above, opposite in direction.

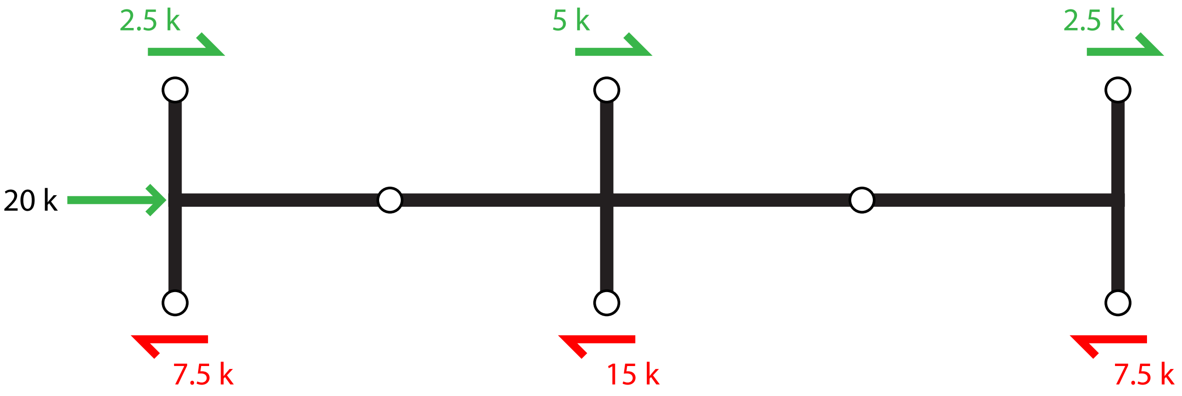


$\sum F = 0$, so $20\text{ k} + 2.5\text{ k} + 5\text{ k} + 2.5\text{ k} - V - 2V - V = 0$

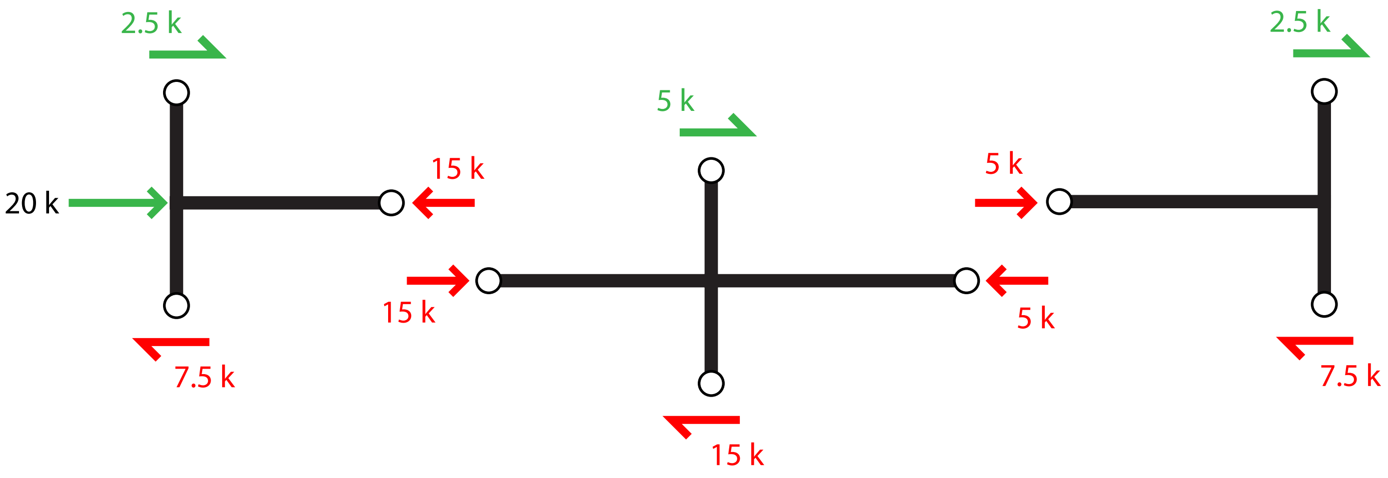
$30\text{ k} - 4V = 0$

$V = 7.5\text{ k}$

This results in the following equilibrium:



Divide this into three assemblies as before in preparation to calculate moments and axial forces.



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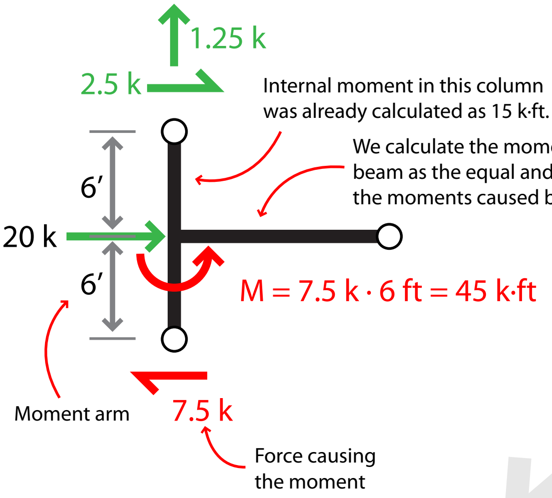
CALCULATE MOMENTS...

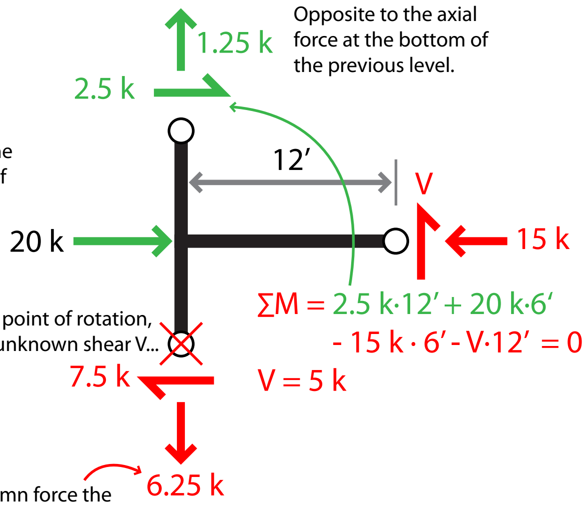
...CALCULATE RESULTING SHEARS AND AXIAL FORCES



Internal moment in this column was already calculated as 15 k·ft.

We calculate the moment at the left end of the beam as the equal and opposite of the sum of the moments caused by the columns.

Force causing the moment



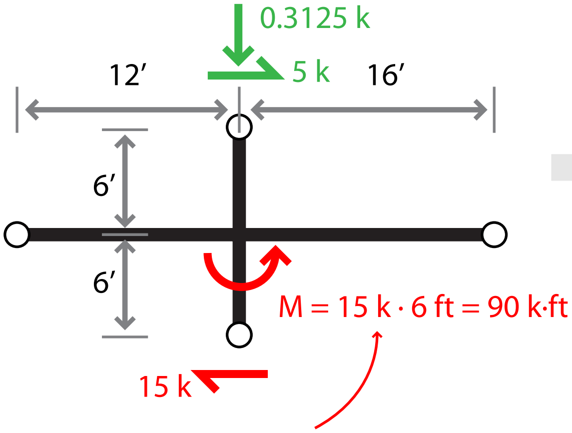
Opposite to the axial force at the bottom of the previous level.

$$\Sigma M = 2.5\text{ k}\cdot 12' + 20\text{ k}\cdot 6' - 15\text{ k}\cdot 6' - V\cdot 12' = 0$$

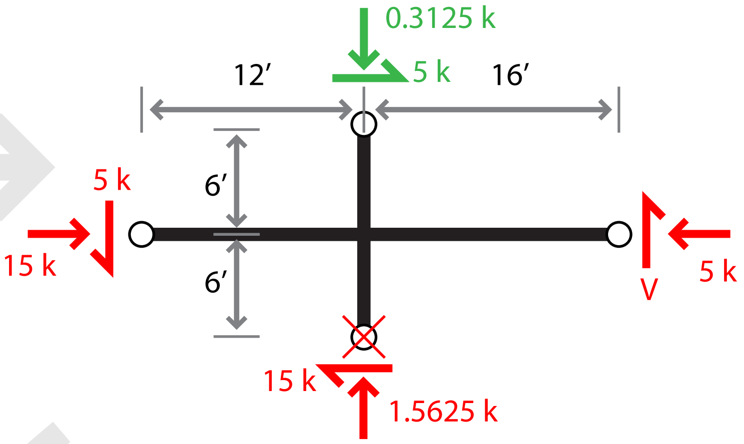
$$V = 5\text{ k}$$

With this new point of rotation, calculate the unknown shear V...

Axial column force the opposes the one above and the shear V.



Note that this is the internal bending moment at the end of the column, not the moment caused by all the forces around the assembly.



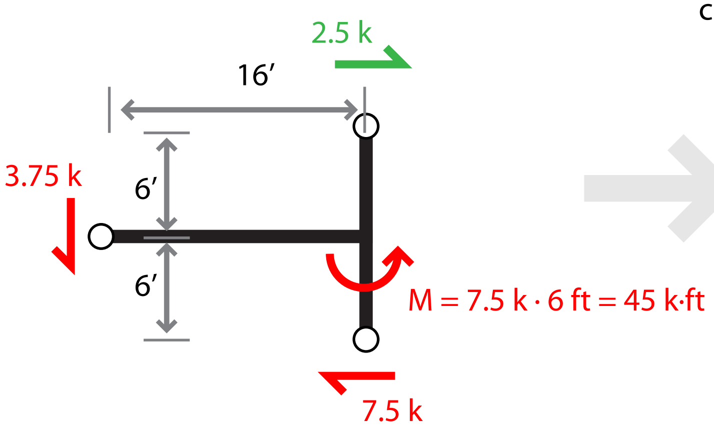
Set up the equilibrium to calculate V:

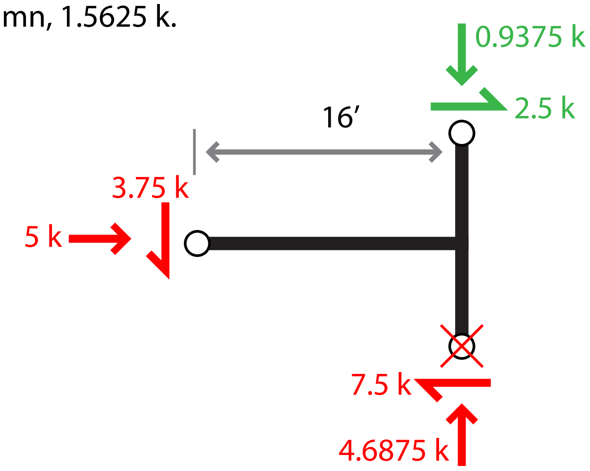
$$\Sigma M = (5\text{ k})(12') + (15\text{ k})(6') - (5\text{ k})(6') - (5\text{ k})(12') - V(16')$$

$$= 60\text{ k}\cdot\text{ft} + 90\text{ k}\cdot\text{ft} - 30\text{ k}\cdot\text{ft} - 60\text{ k}\cdot\text{ft} - 16V = 0$$

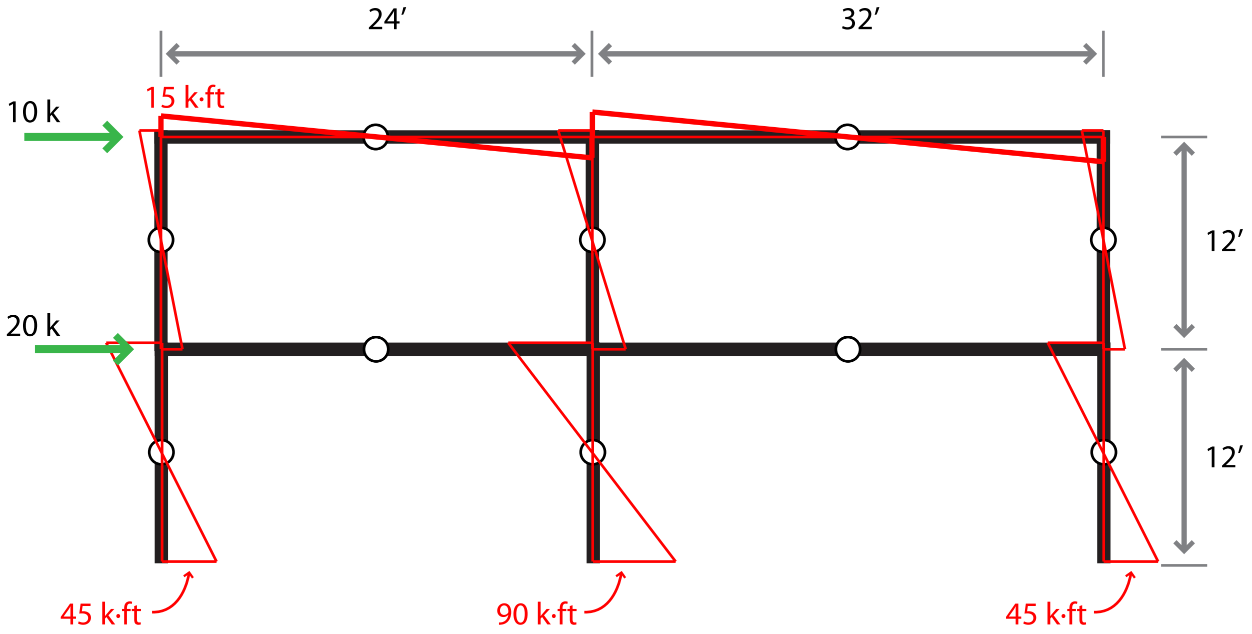
$$V = 3.75\text{ k}$$

After we have V, we can calculate the axial load in the column, 1.5625 k.

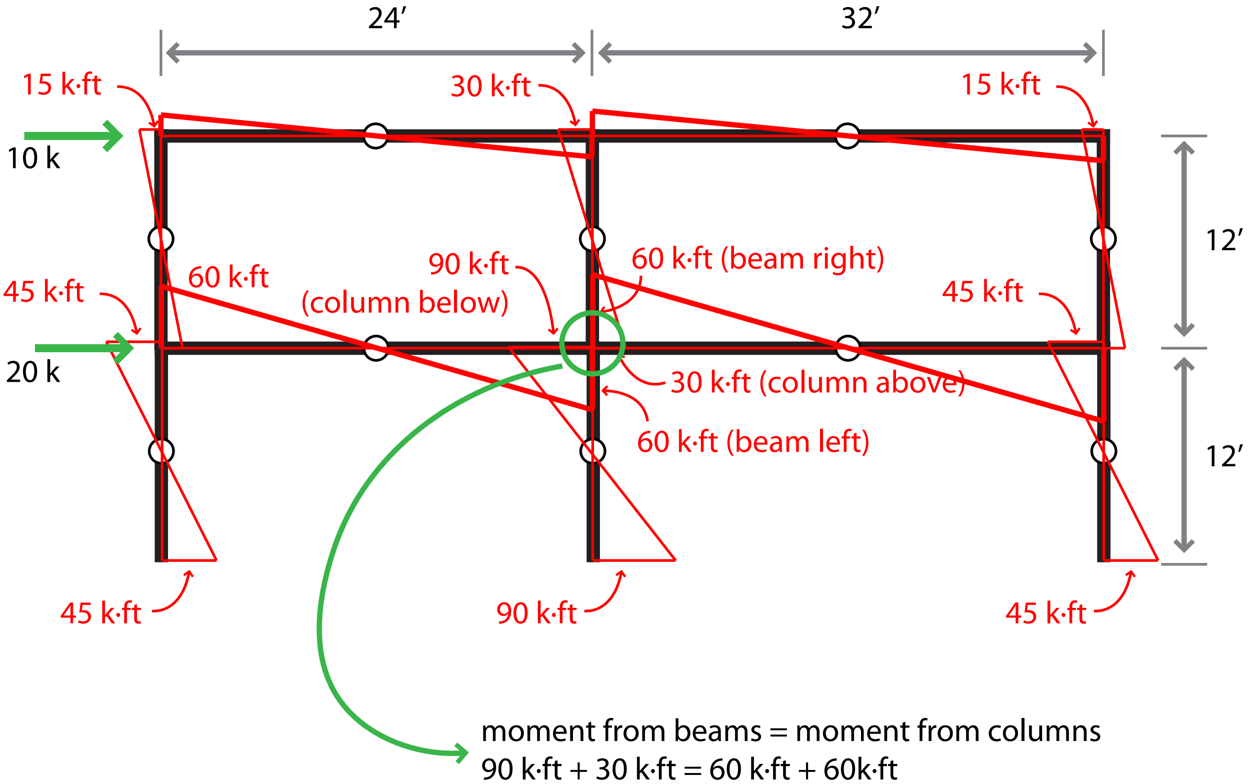




Given the moments just calculated, we can now graph the moment diagrams on the columns:

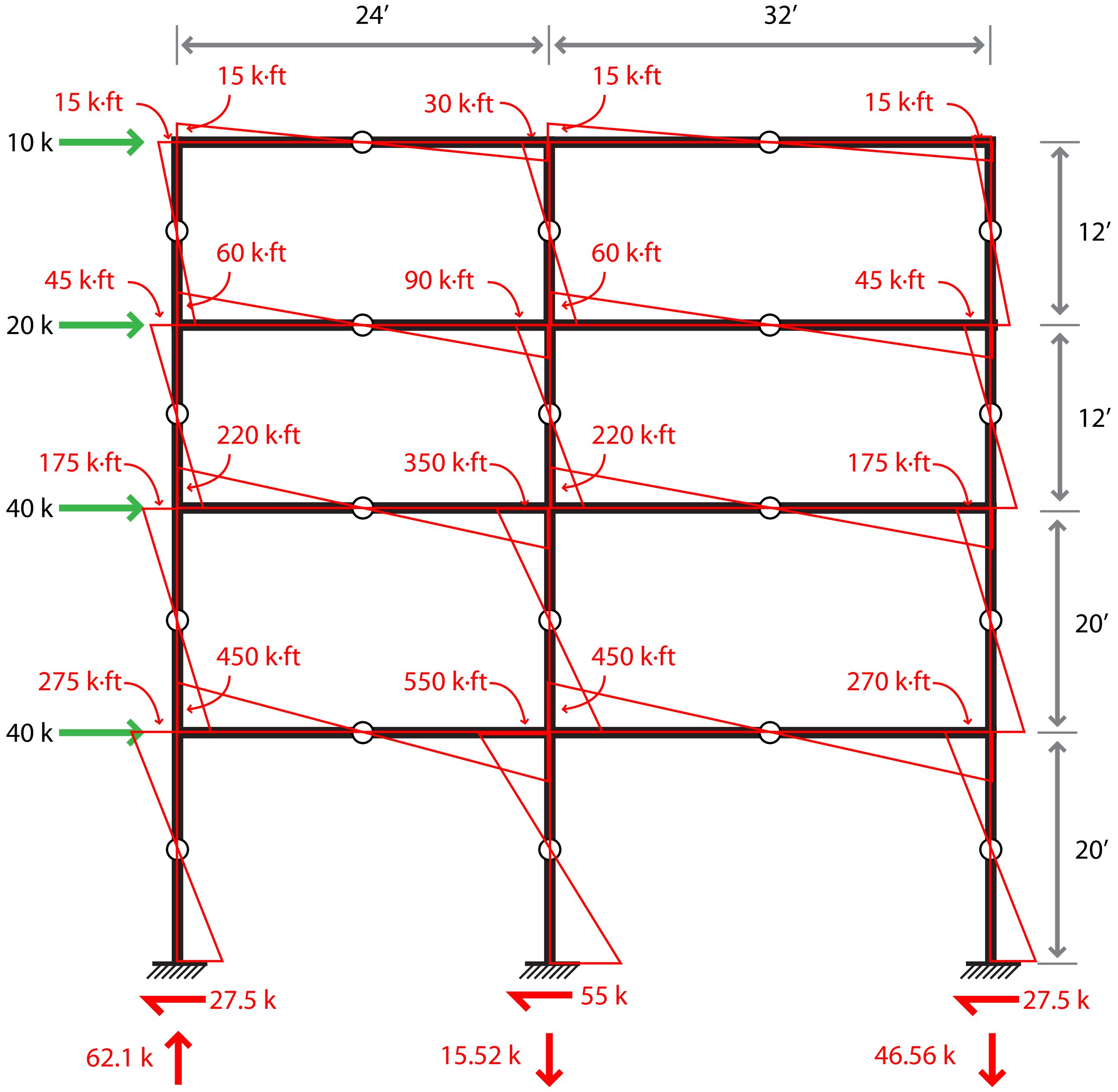


Moment diagrams for the beams come from the moment values at the columns. Add the moments across the joint to get the moments required for the beams across the same joint.



moment from beams = moment from columns
90 k·ft + 30 k·ft = 60 k·ft + 60k·ft

We can continue to work our way down the frame with this method...

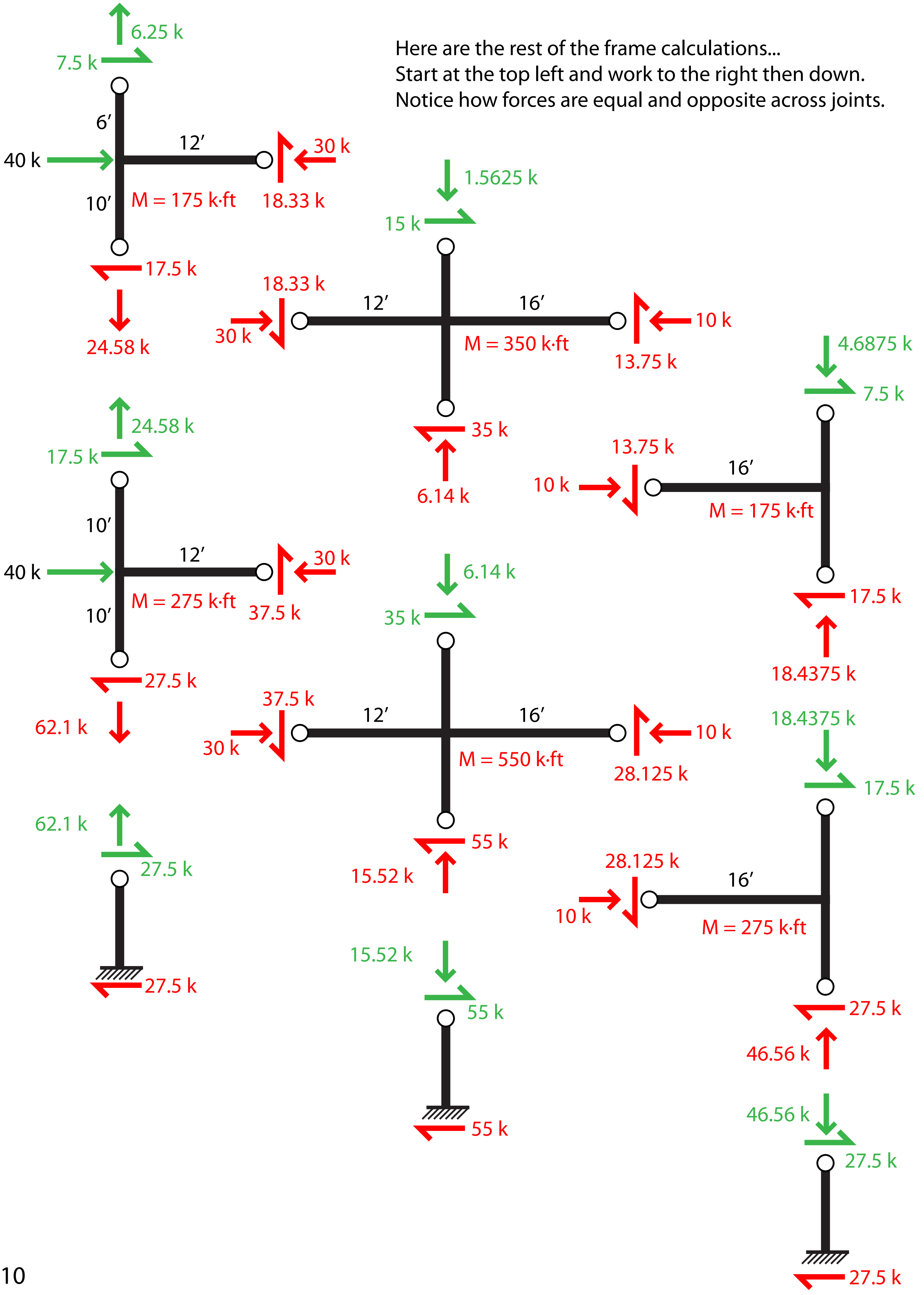


...until we get the base shear and axial reactions at the foundation supports.

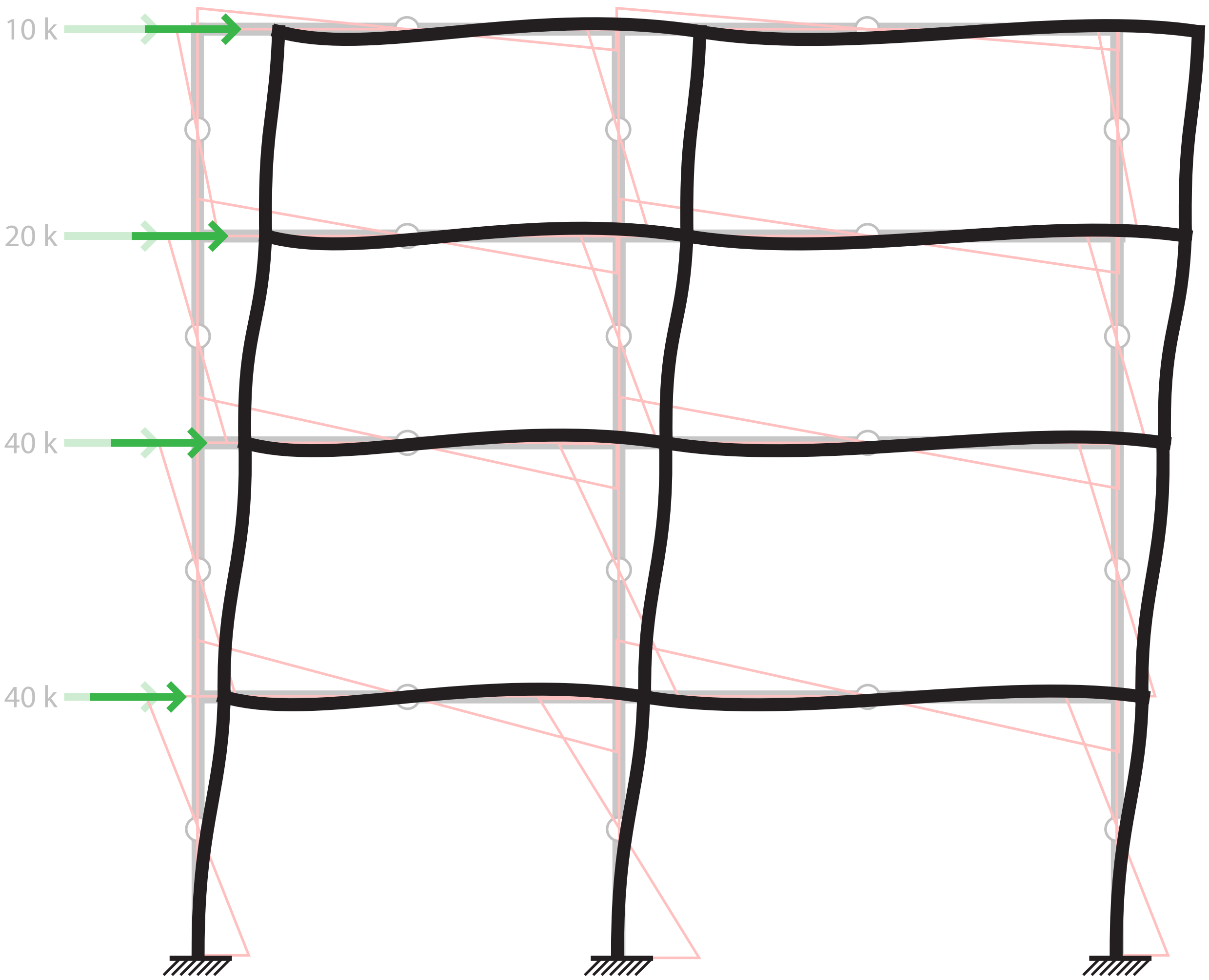
Moments for the beams are determined by summing the moments of the columns at the leftmost joint first. Then work from left to right.

The moment diagrams appear on the *compression side* of the given column or beam.

Here are the rest of the frame calculations...
Start at the top left and work to the right then down.
Notice how forces are equal and opposite across joints.



Deflections and moment are closely related. By convention, we place the moment diagrams on the compression sides of the beams and columns. Those sides naturally get shorter and thus curve inwards toward the diagram.



The joints themselves will translate to the right, in the direction of the lateral forces. Rigid joints attempt to maintain their 90-degree angles, and inflection points indicate a change in curvature.