



Bending and Shear in Simple Beams

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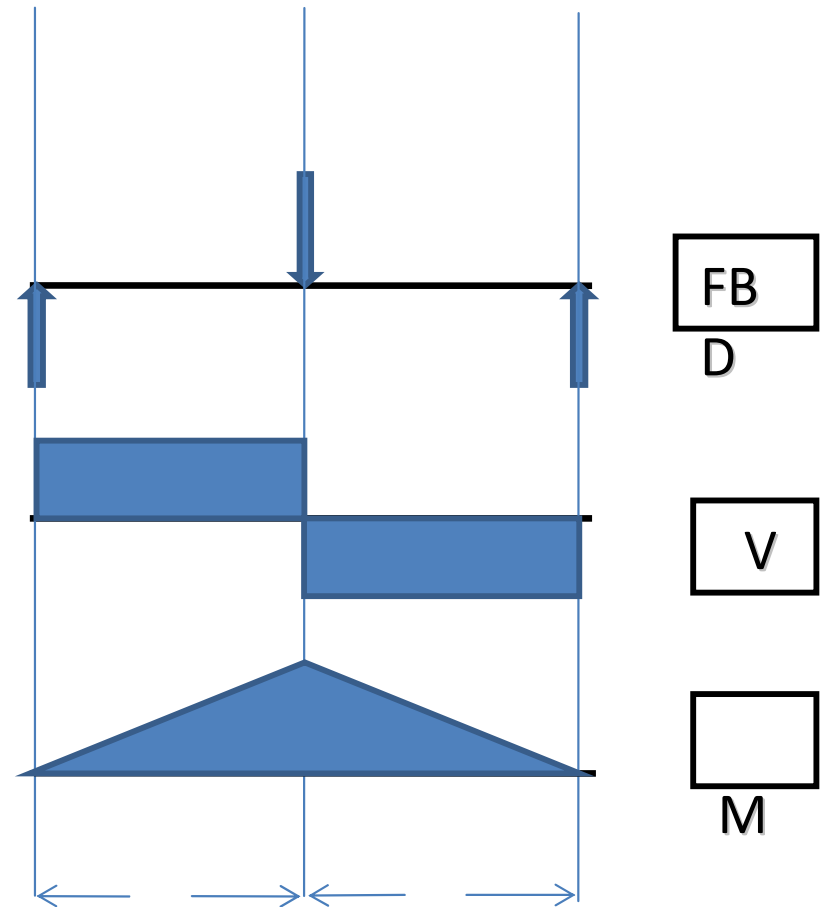
Beams

- Beams resist loads usually applied perpendicular to its longitudinal axis.
- These loads cause bending, which in turn causes internal stresses as it resists this load.

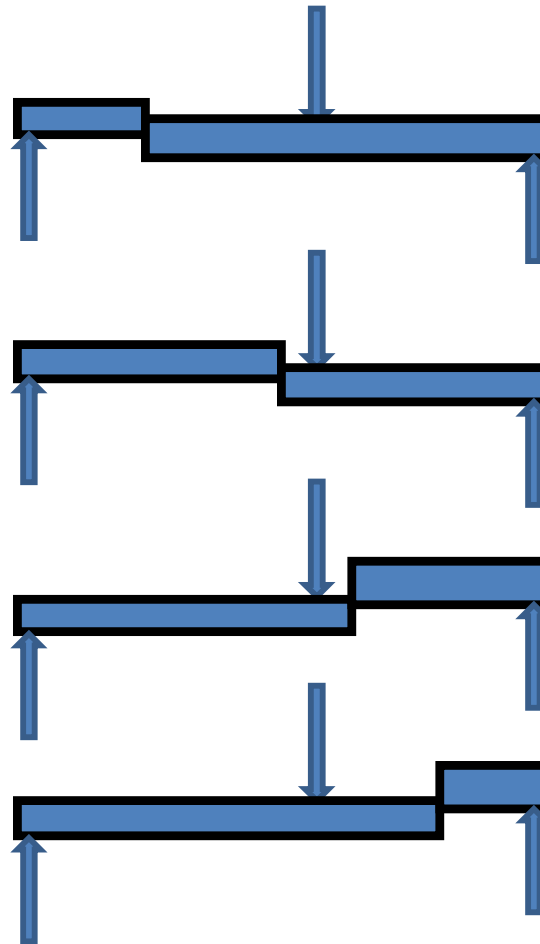


Moment and Shear Diagrams

- Graphs that show the magnitude and direction of the internal shear and moment forces along the length of a beam

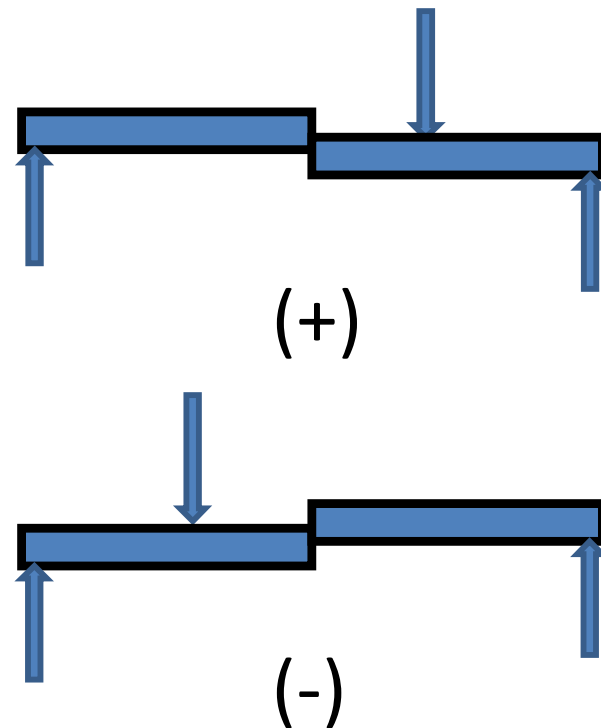
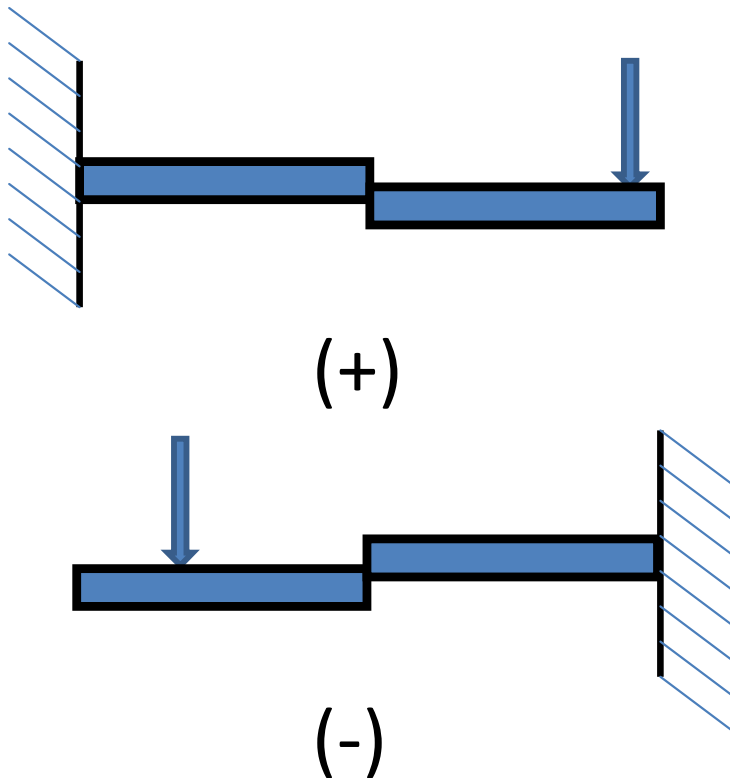


Sign Convention, Shear

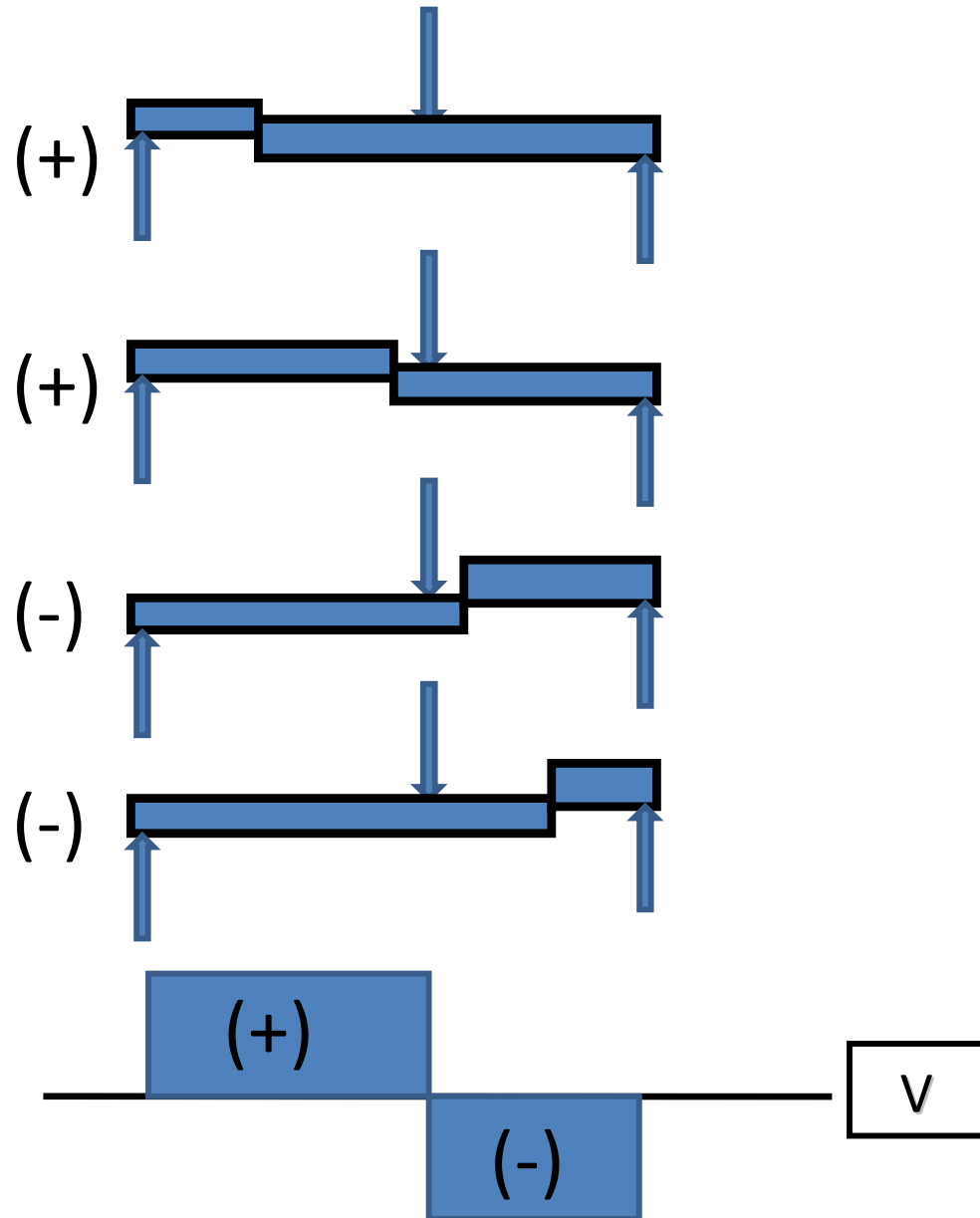


Sign Convention, Shear

- Shear at a section is considered positive (+) when the portion of the beam to the left of the section cut tends to be in the up position with respect to the section in the right

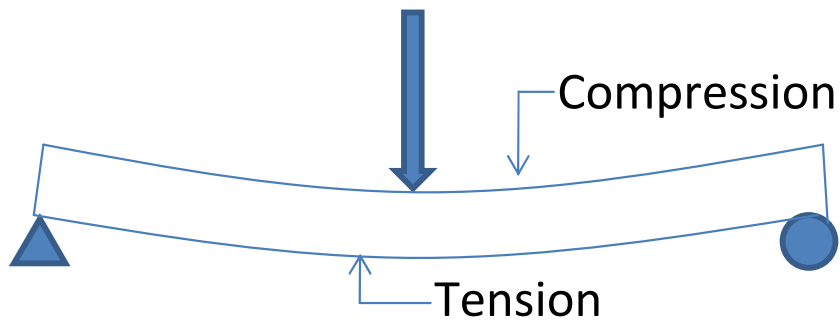


Sign Convention, Shear

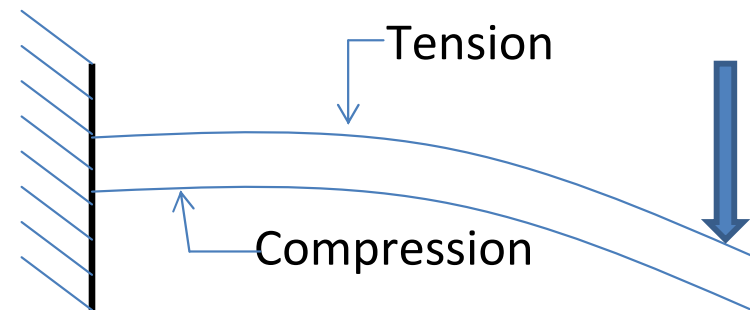


Sign Convention, Moment

- Positive (+) moment occurs at sections for which the top fibers of the beam are in compression and the bottom fibers in tension.



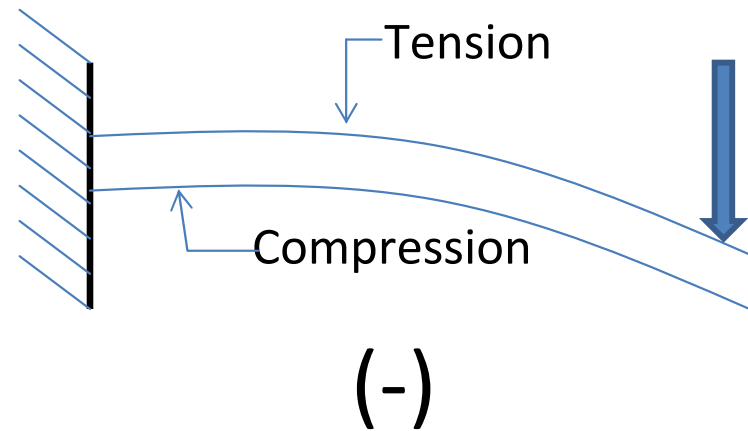
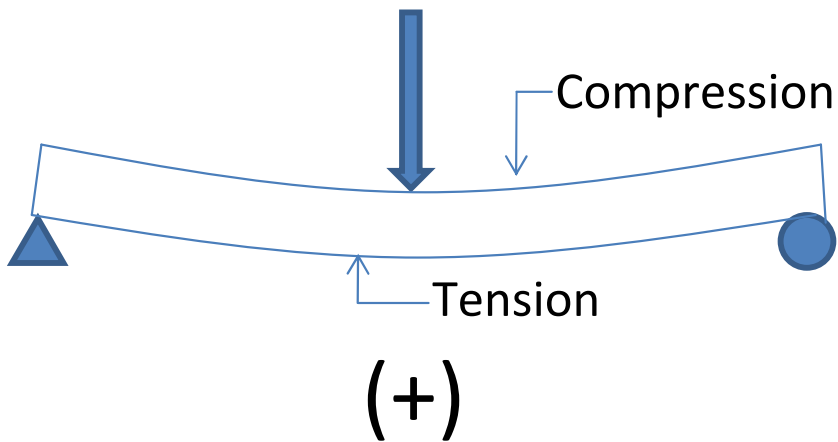
(+)



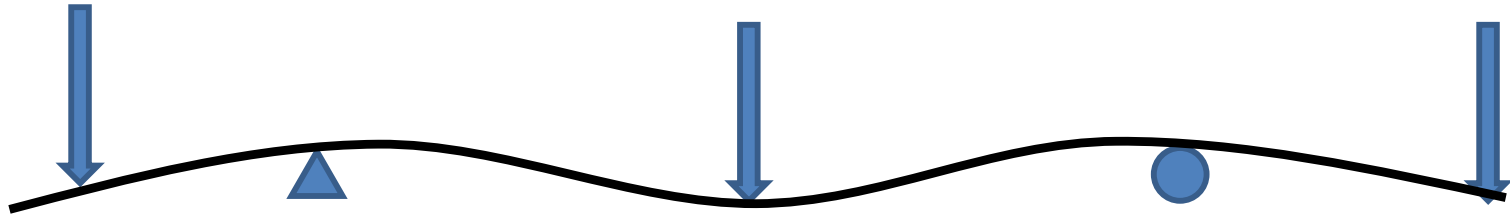
(-)

Sign Convention, Moment

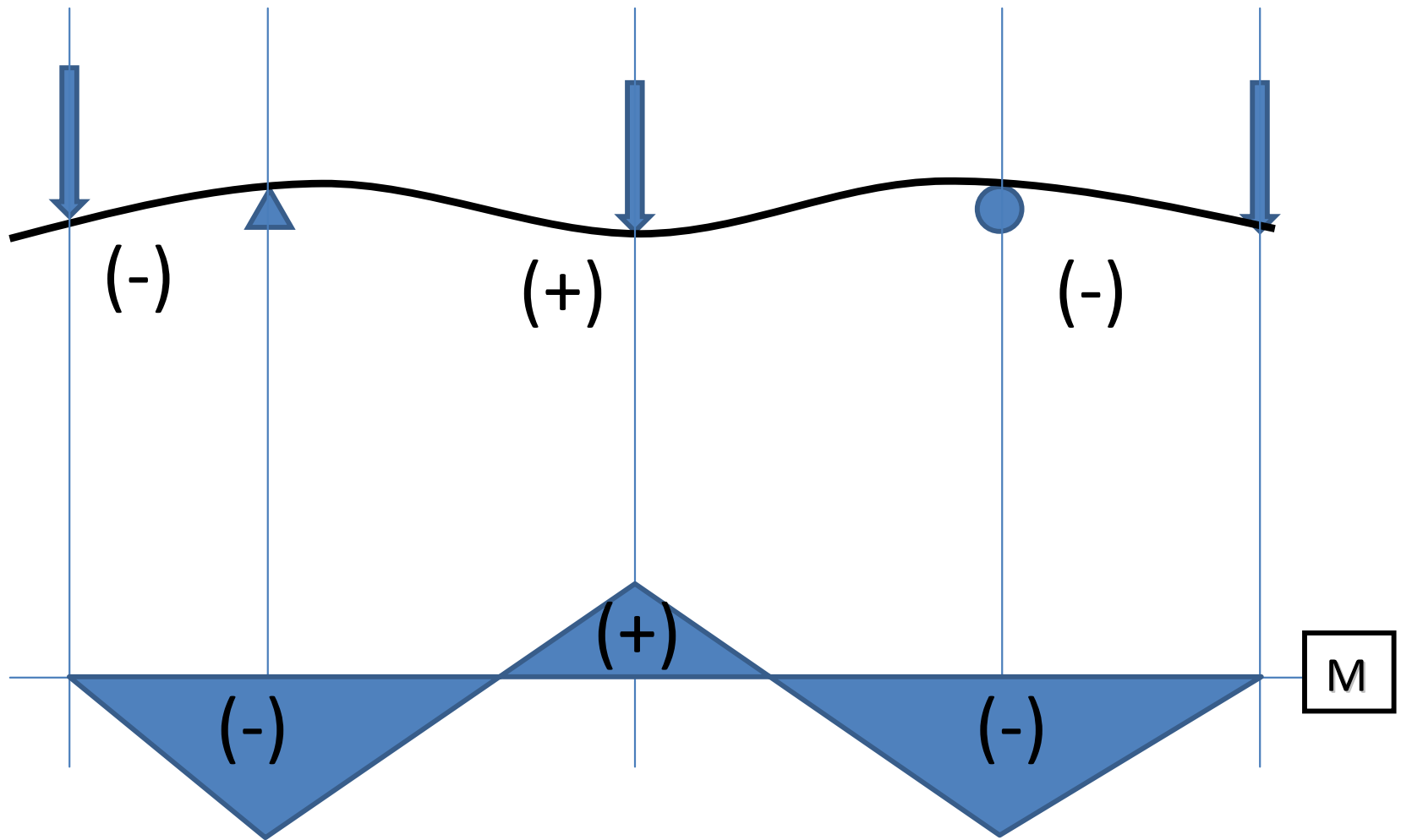
- Positive (+) moment occurs at sections for which the top fibers of the beam are in compression and the bottom fibers in tension.



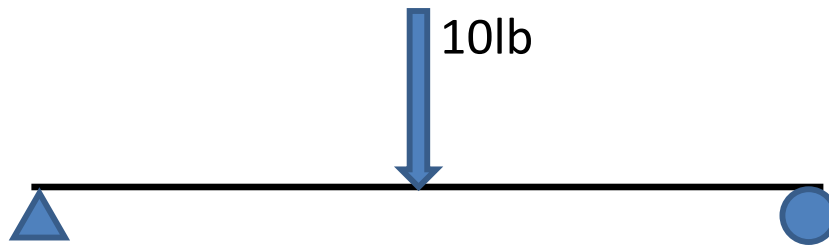
Sign Convention, Moment



Sign Convention, Moment



Constructing Moment and Shear Diagrams



$$\Sigma M_a=0$$

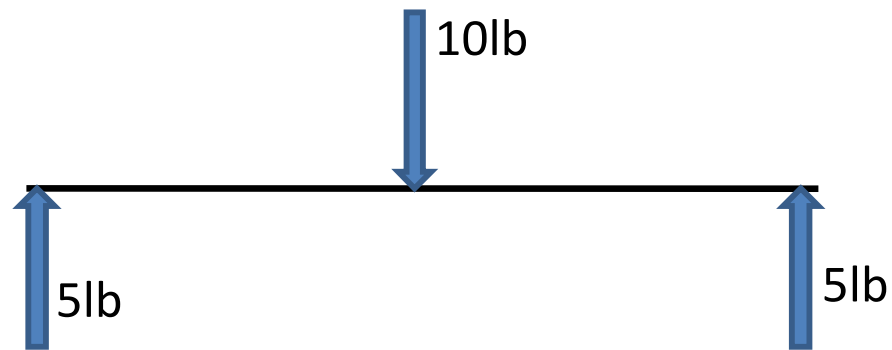
$$-(10\text{lb})(1\text{ft})+(R_{y_2})(2\text{ft})=0$$

$$R_{y_2}=5\text{lb}$$

$$\Sigma F_y=0$$

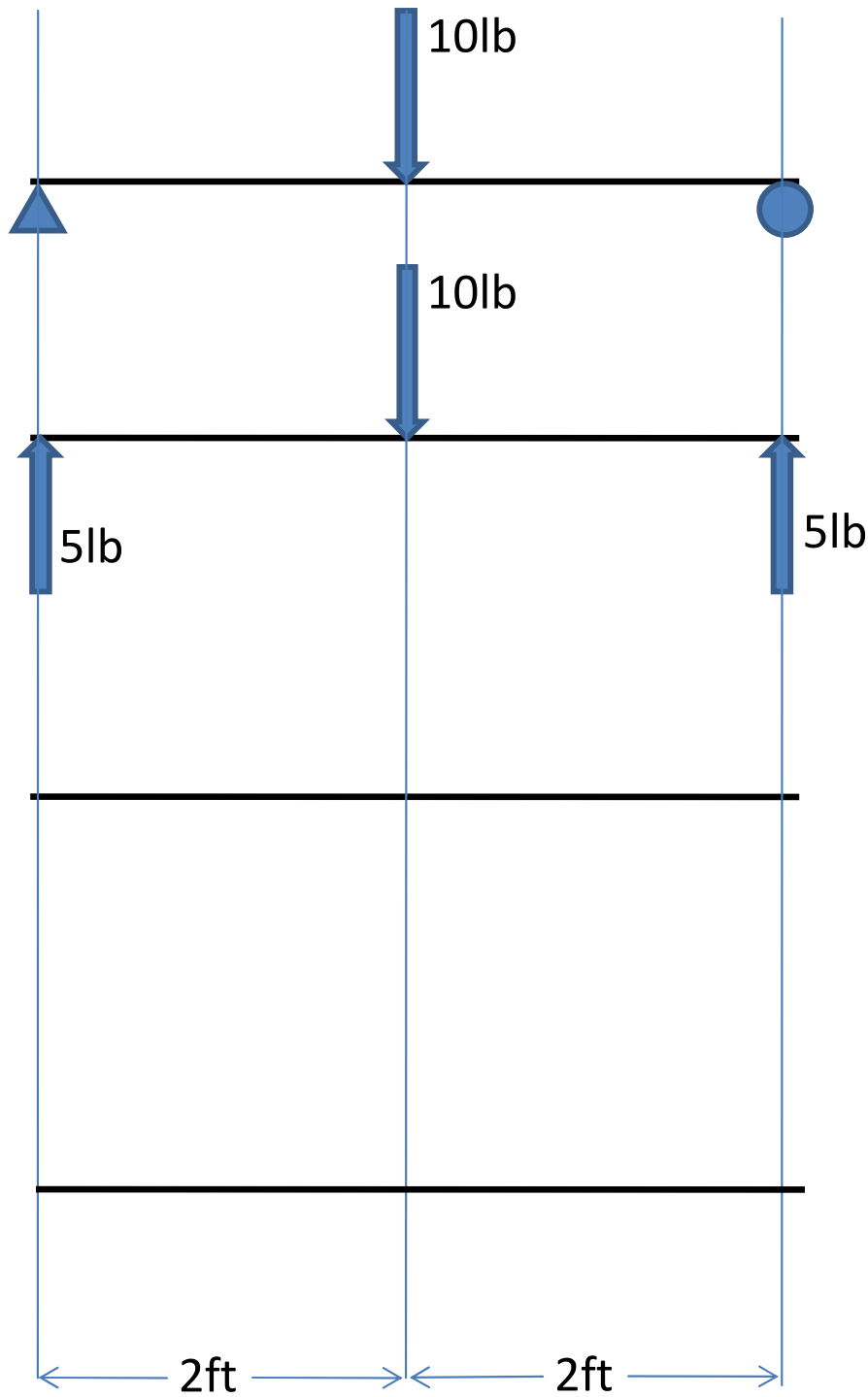
$$R_{y_1}-10\text{lb}+5\text{lb}=0$$

$$R_{y_1}=5\text{lb}$$



FB

D

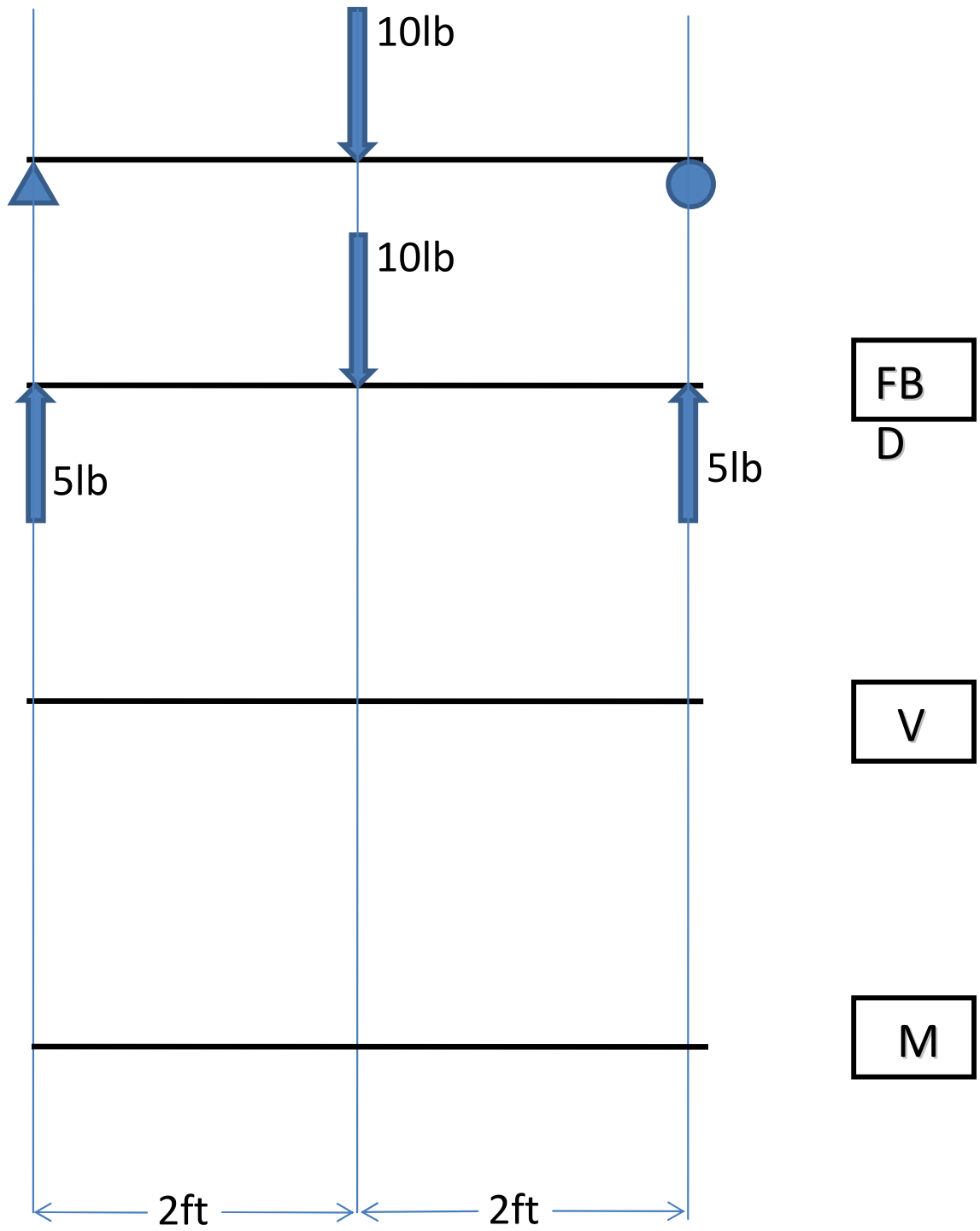


FB

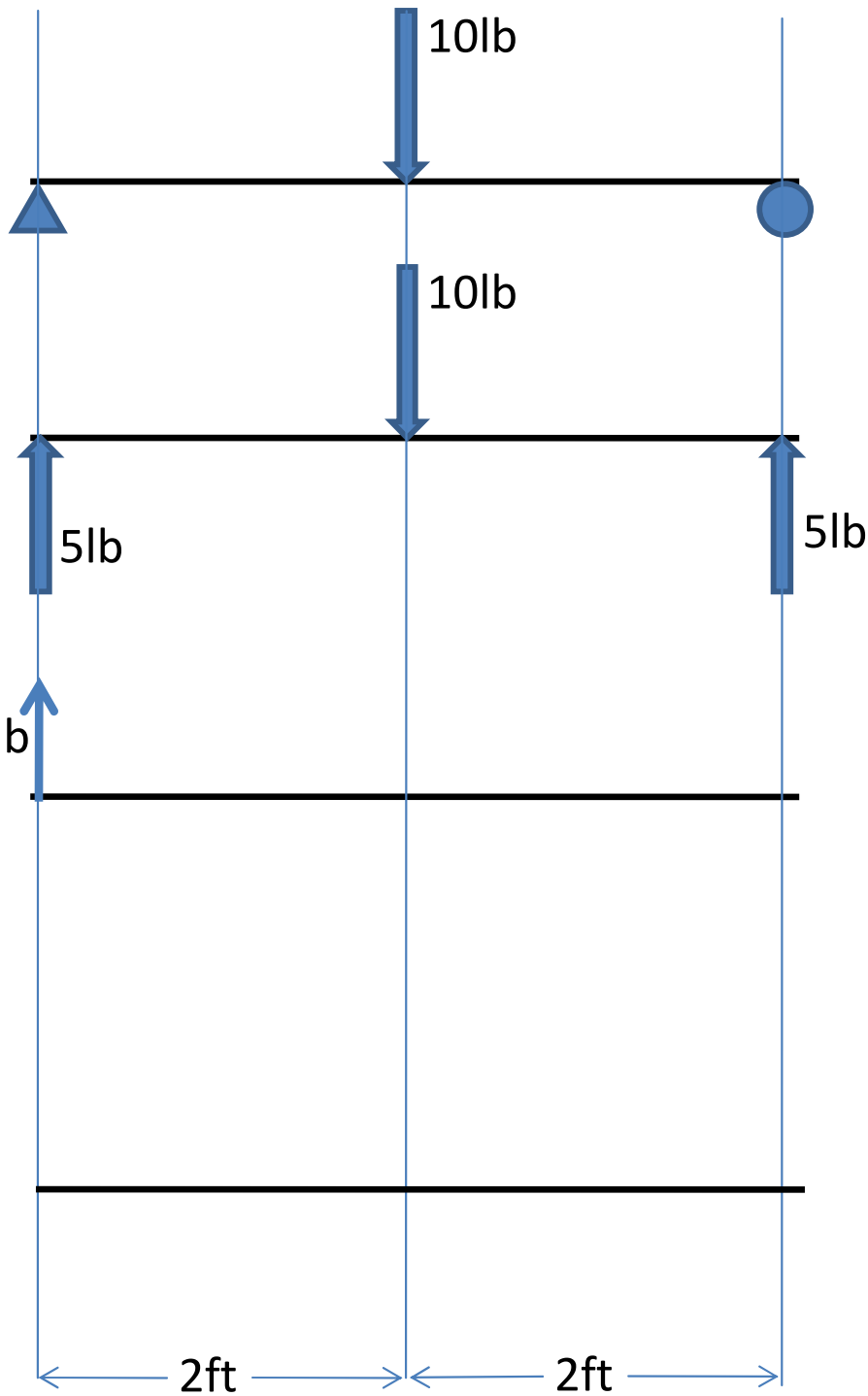
D

V

M



Concentrated loads shift the SHEAR diagram up or down



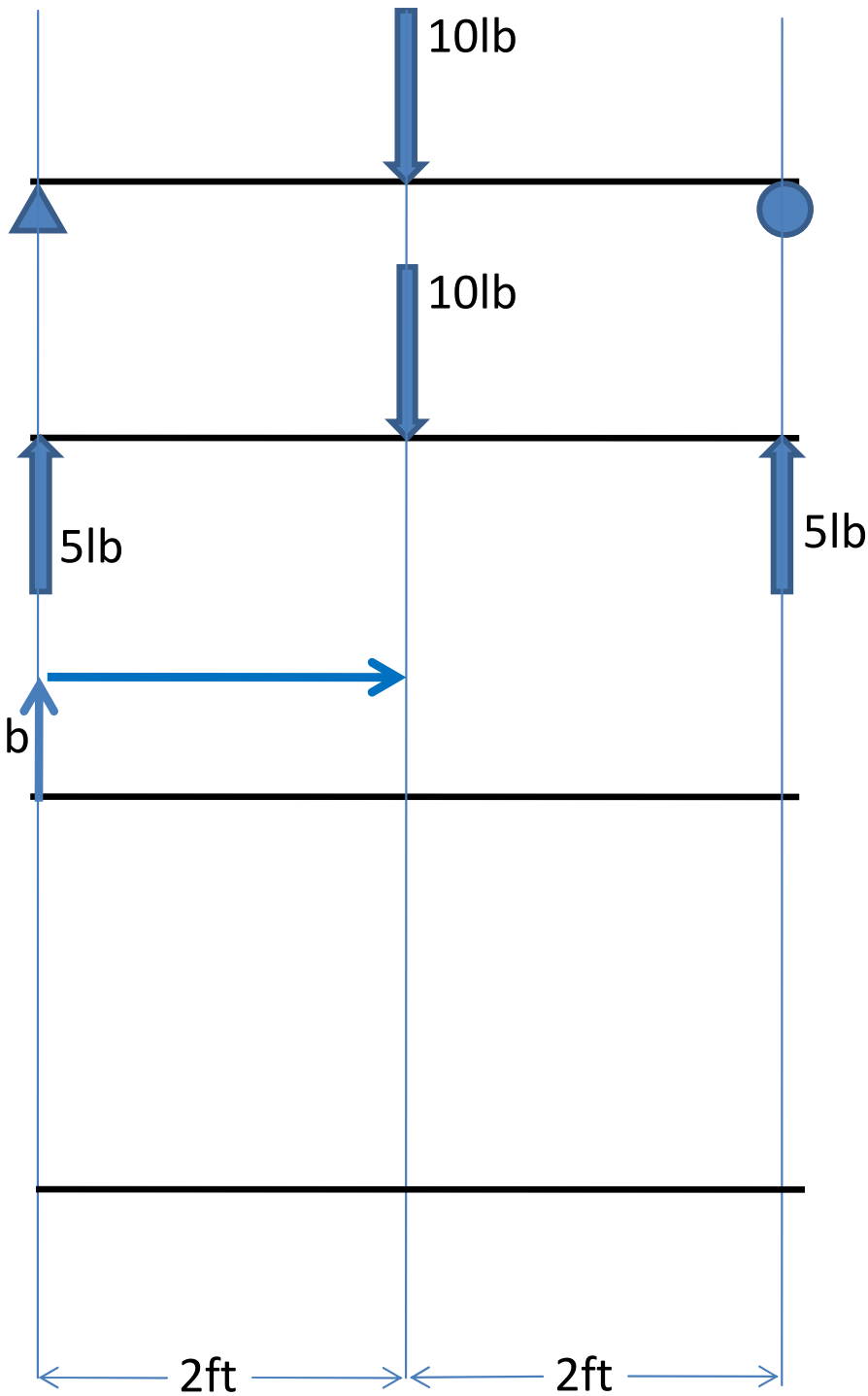
FB

D

V

M

Concentrated loads shift the SHEAR diagram up or down



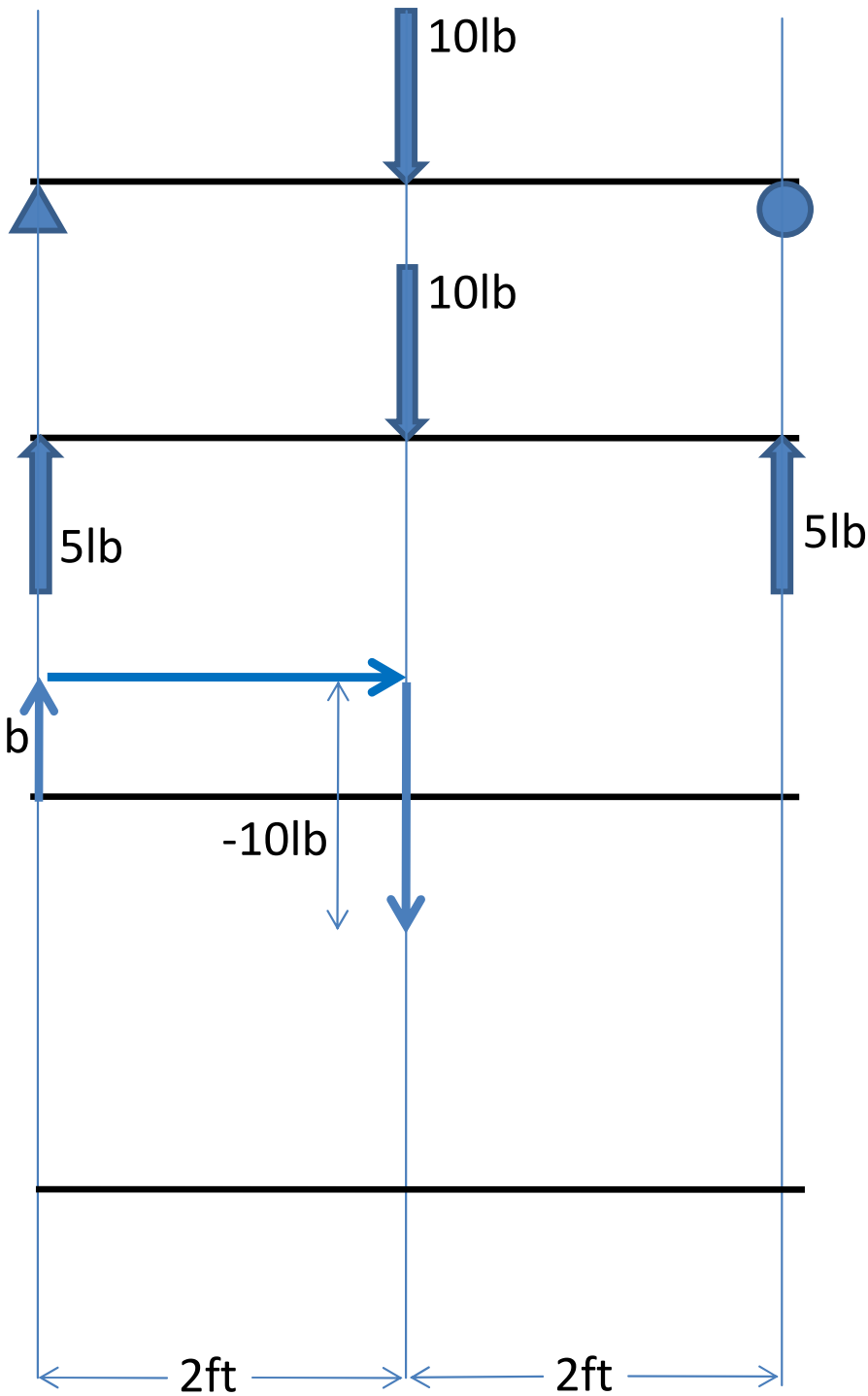
FB

D

V

M

Concentrated loads shift the SHEAR diagram up or down



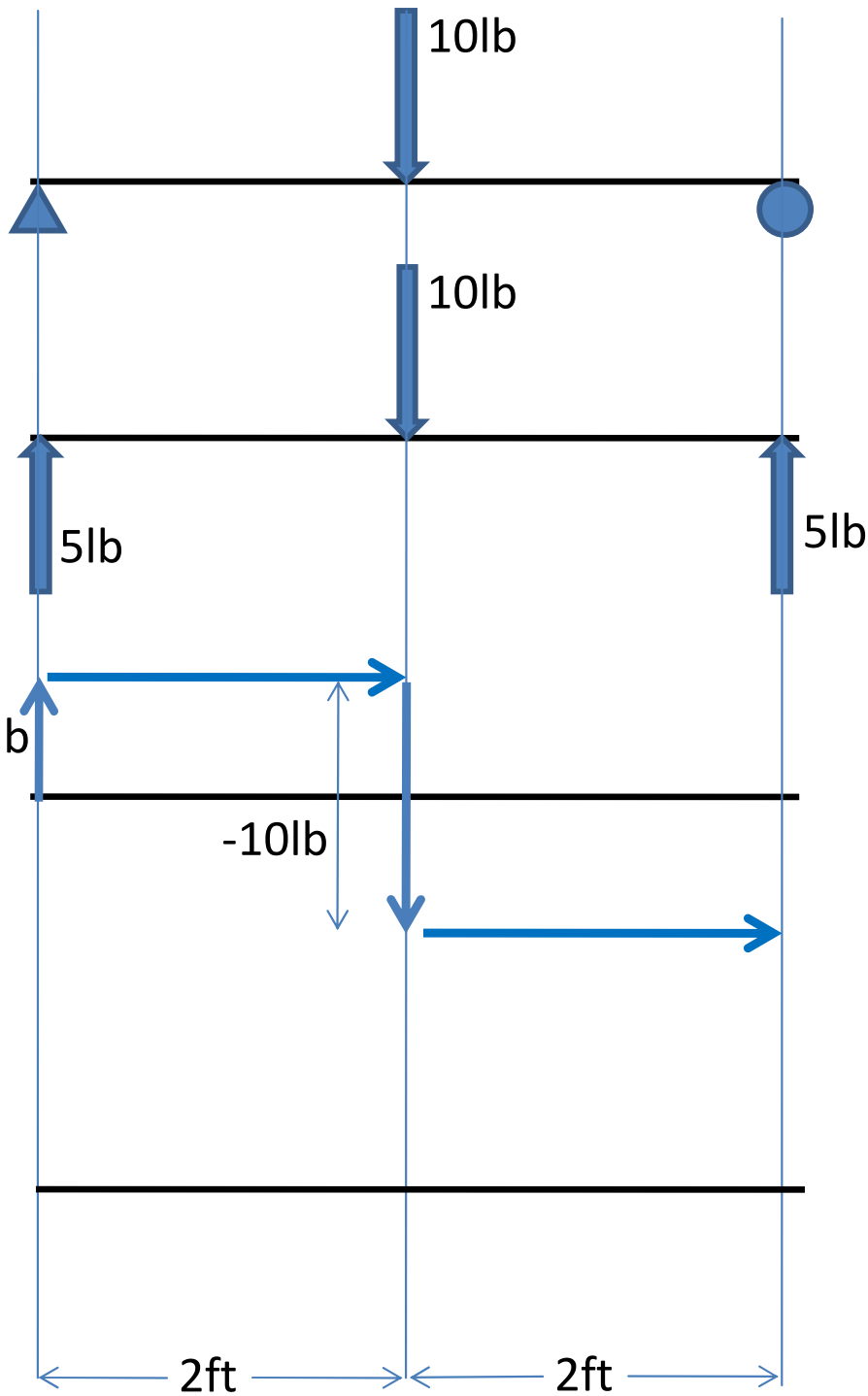
FB

D

V

M

Concentrated loads shift the SHEAR diagram up or down



FB

D

V

M

Concentrated loads shift the SHEAR diagram up or down

$5lb$

$5lb$

$10lb$

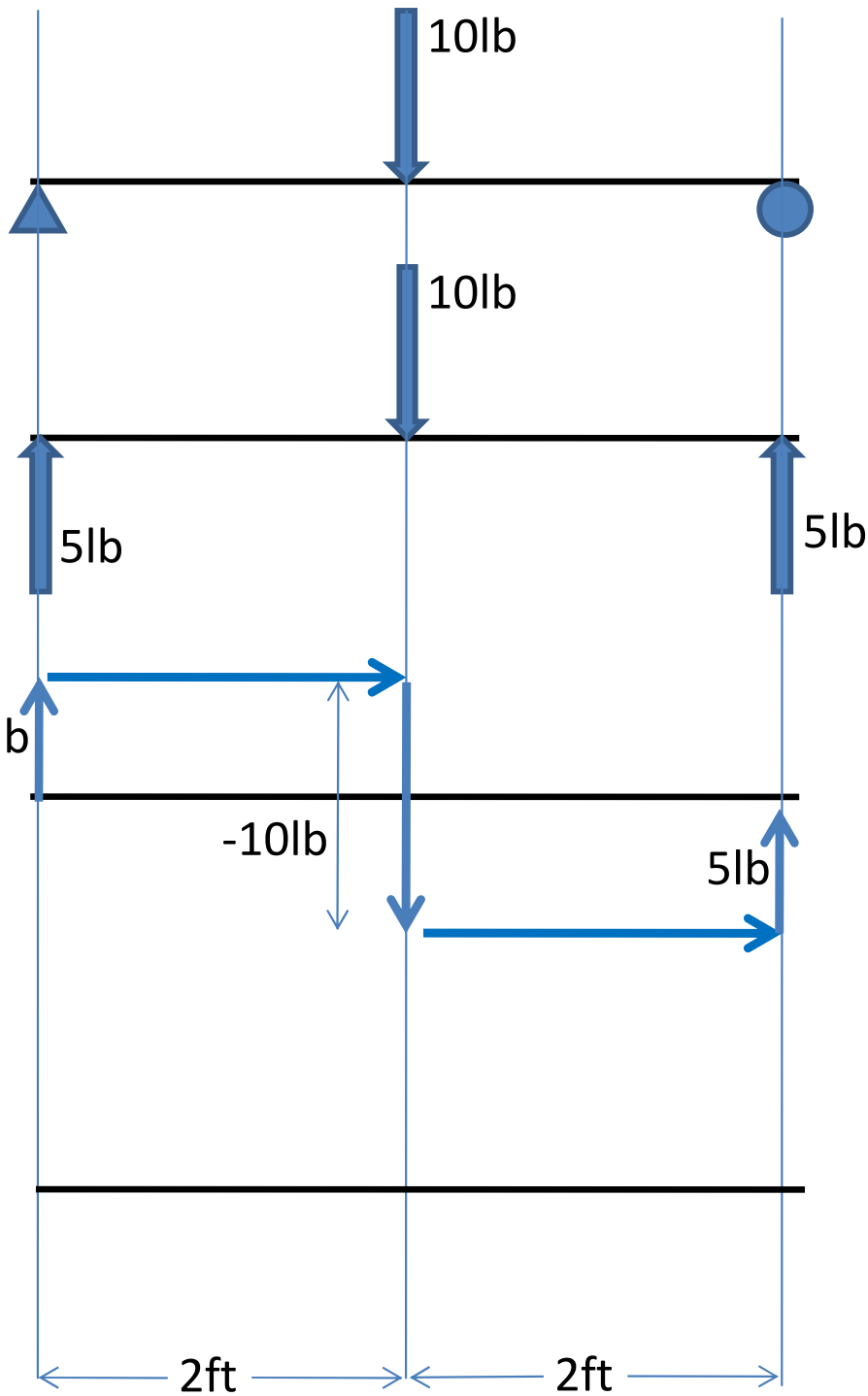
$10lb$

$5lb$

$-10lb$

$2ft$

$2ft$



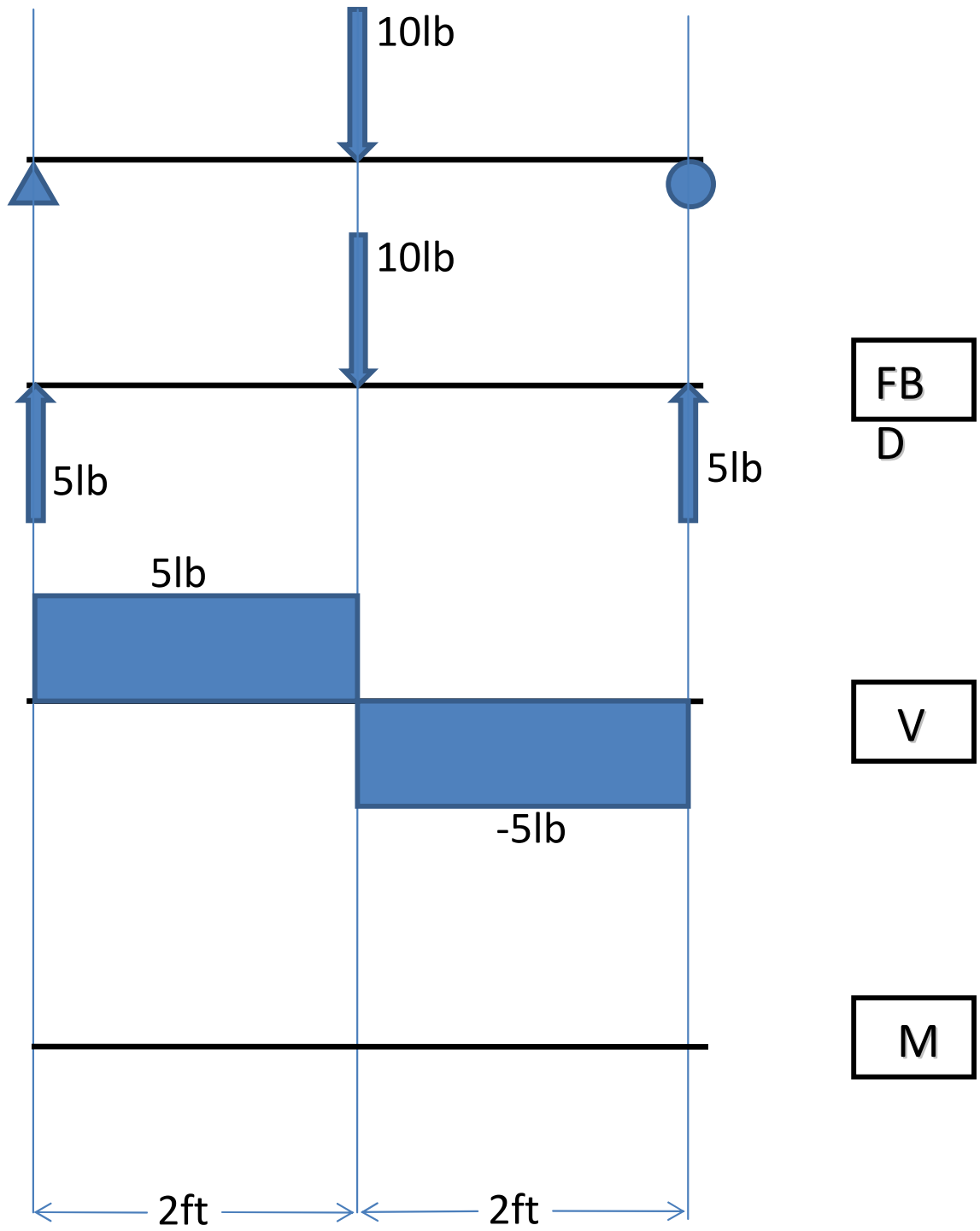
Concentrated loads shift the SHEAR diagram up or down

FB

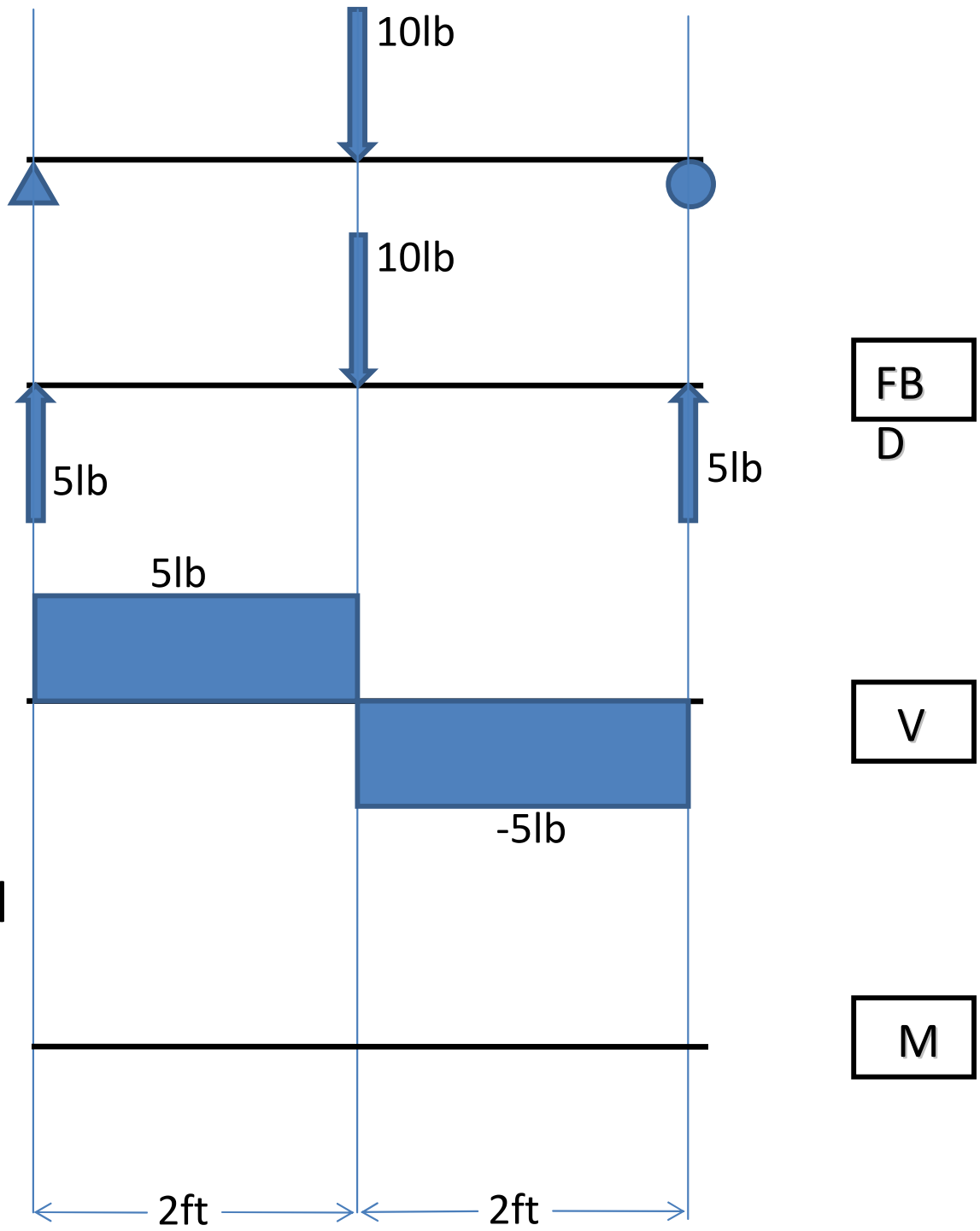
D

V

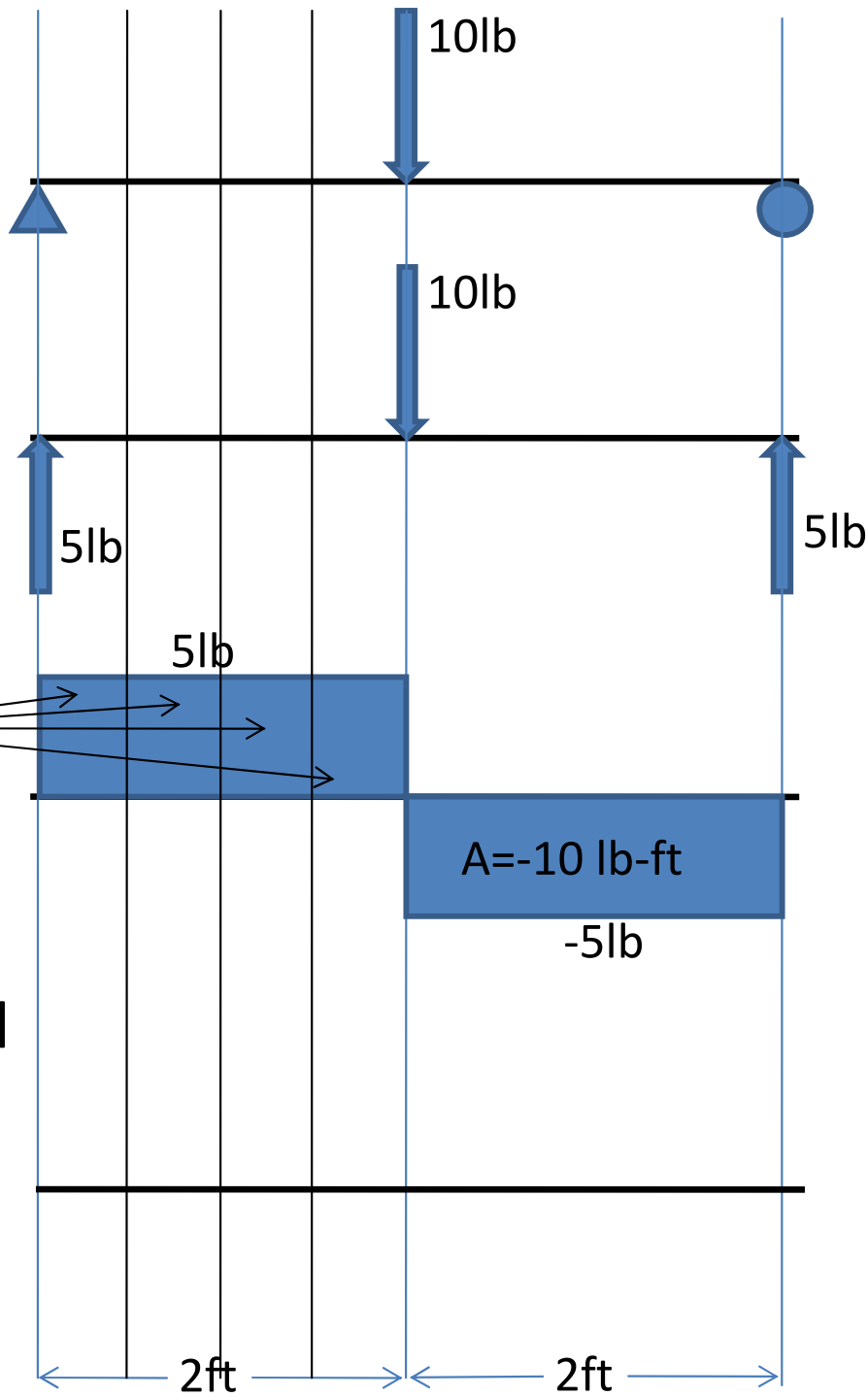
M



Concentrated loads shift the SHEAR diagram up or down



AREA under V
diagram is equal
to the CHANGE
in the M
diagram



FB

D

V

M

$A = 2.5\text{ lb-ft}$

AREA under V diagram is equal to the CHANGE in the M diagram

10 lb

10 lb

5 lb

5 lb

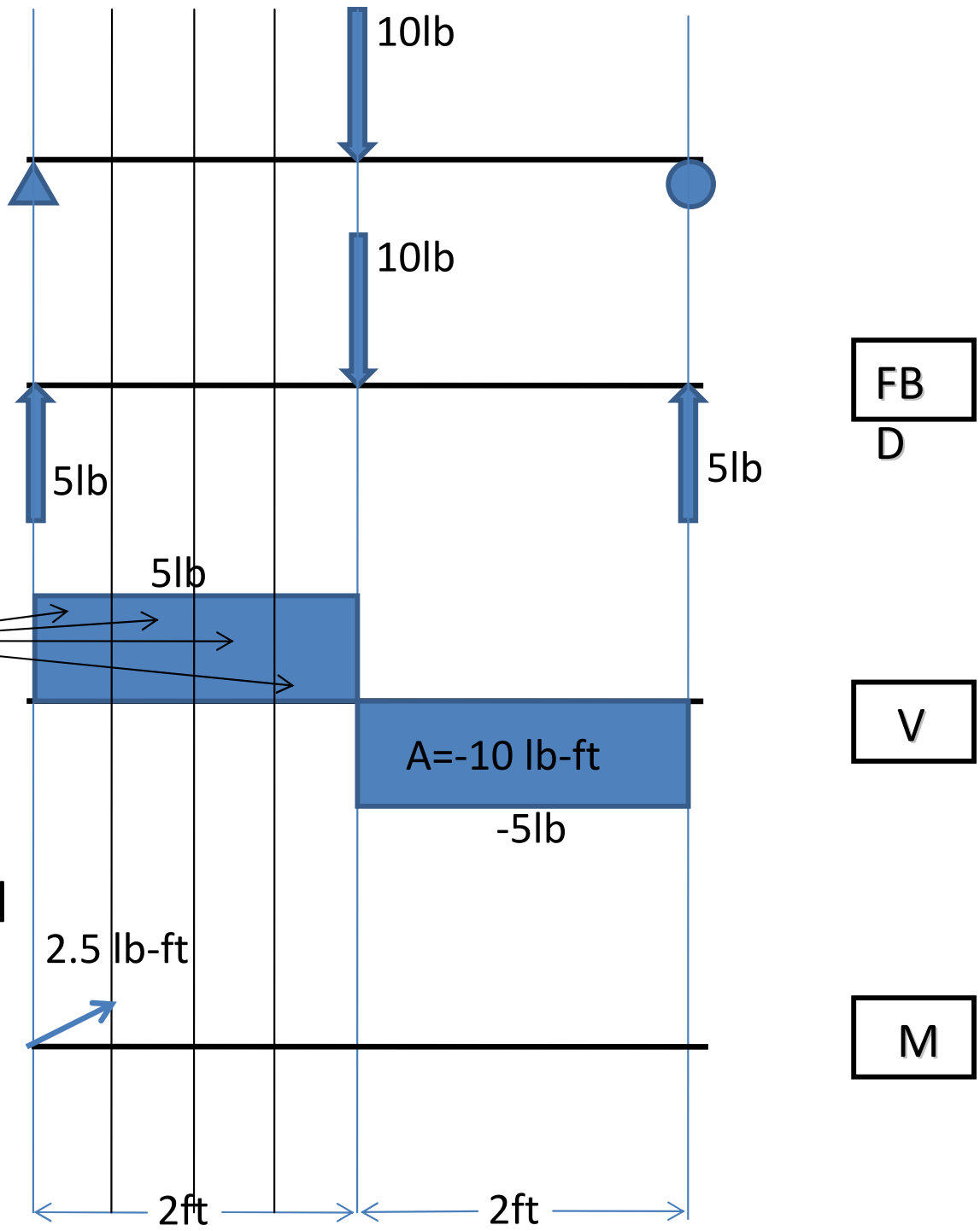
5 lb

$A = -10\text{ lb-ft}$

-5 lb

2 ft

2 ft



$A = 2.5 lb-ft$

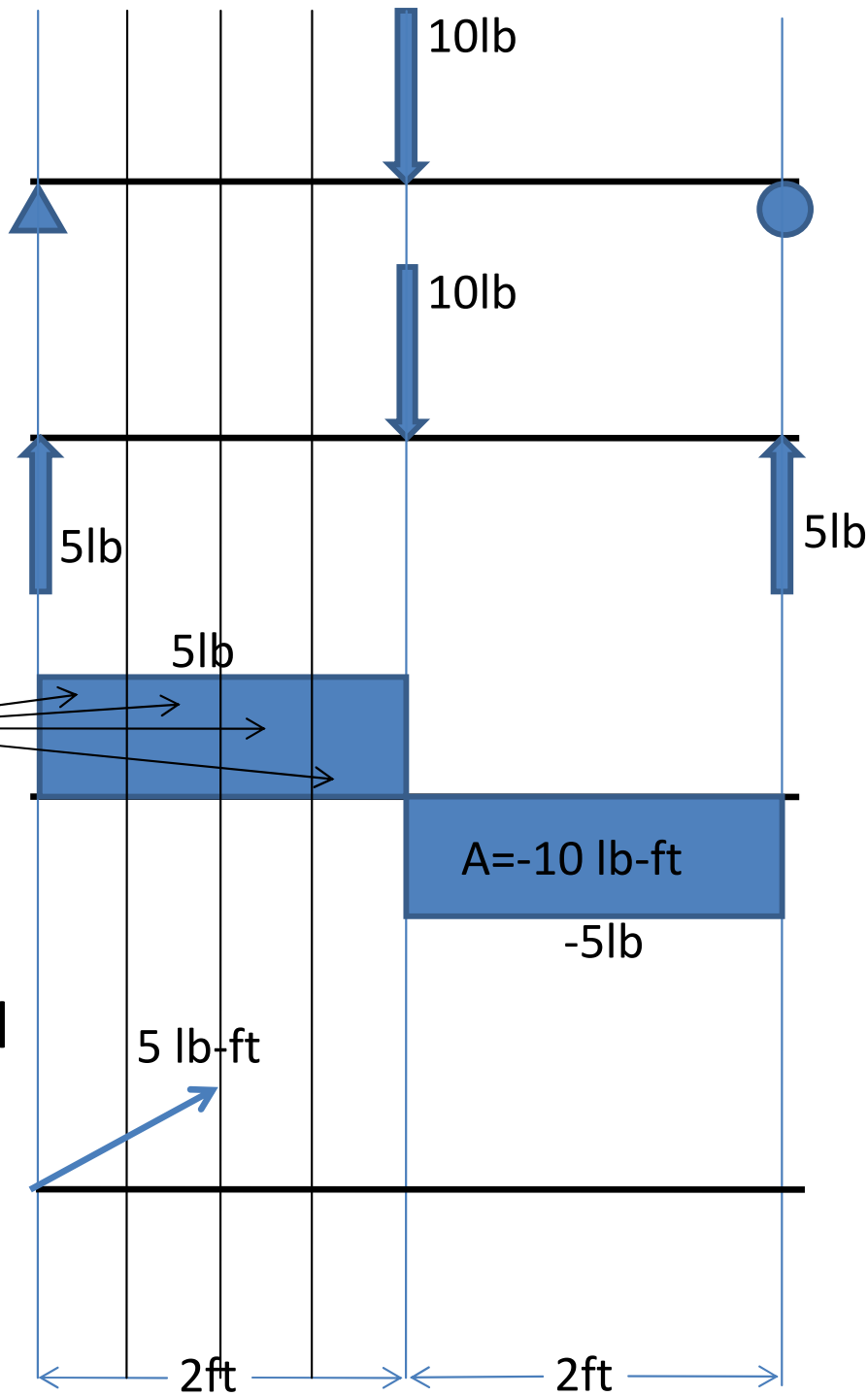
AREA under V diagram is equal to the CHANGE in the M diagram

FB

D

V

M



FB

D

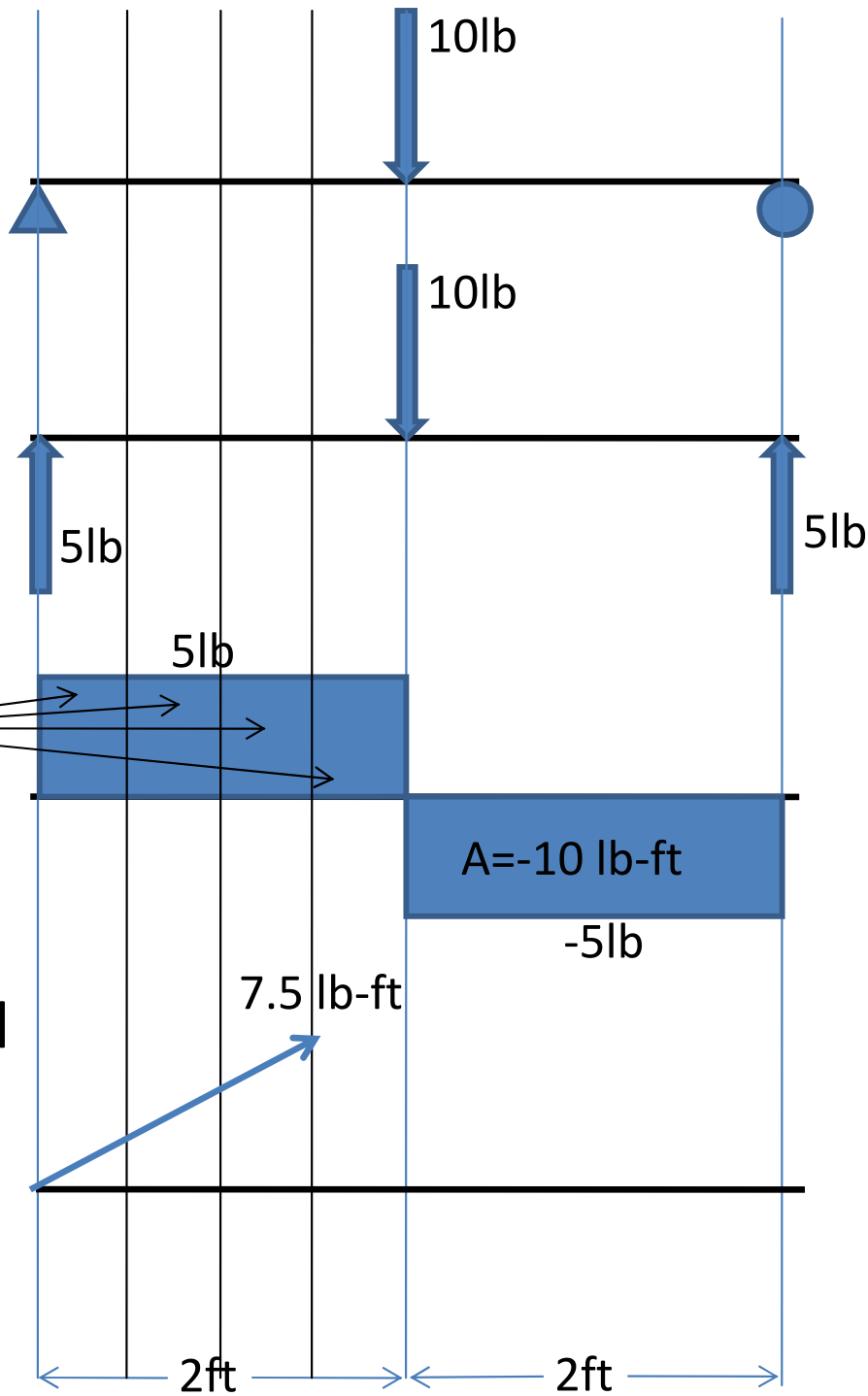
V

M

$A=2.5 \text{ lb-ft}$

$A=-10 \text{ lb-ft}$

AREA under V diagram is equal to the CHANGE in the M diagram



FB

D

V

M

$A=2.5 \text{ lb-ft}$

AREA under V diagram is equal to the CHANGE in the M diagram

10lb

10lb

5lb

5lb

5lb

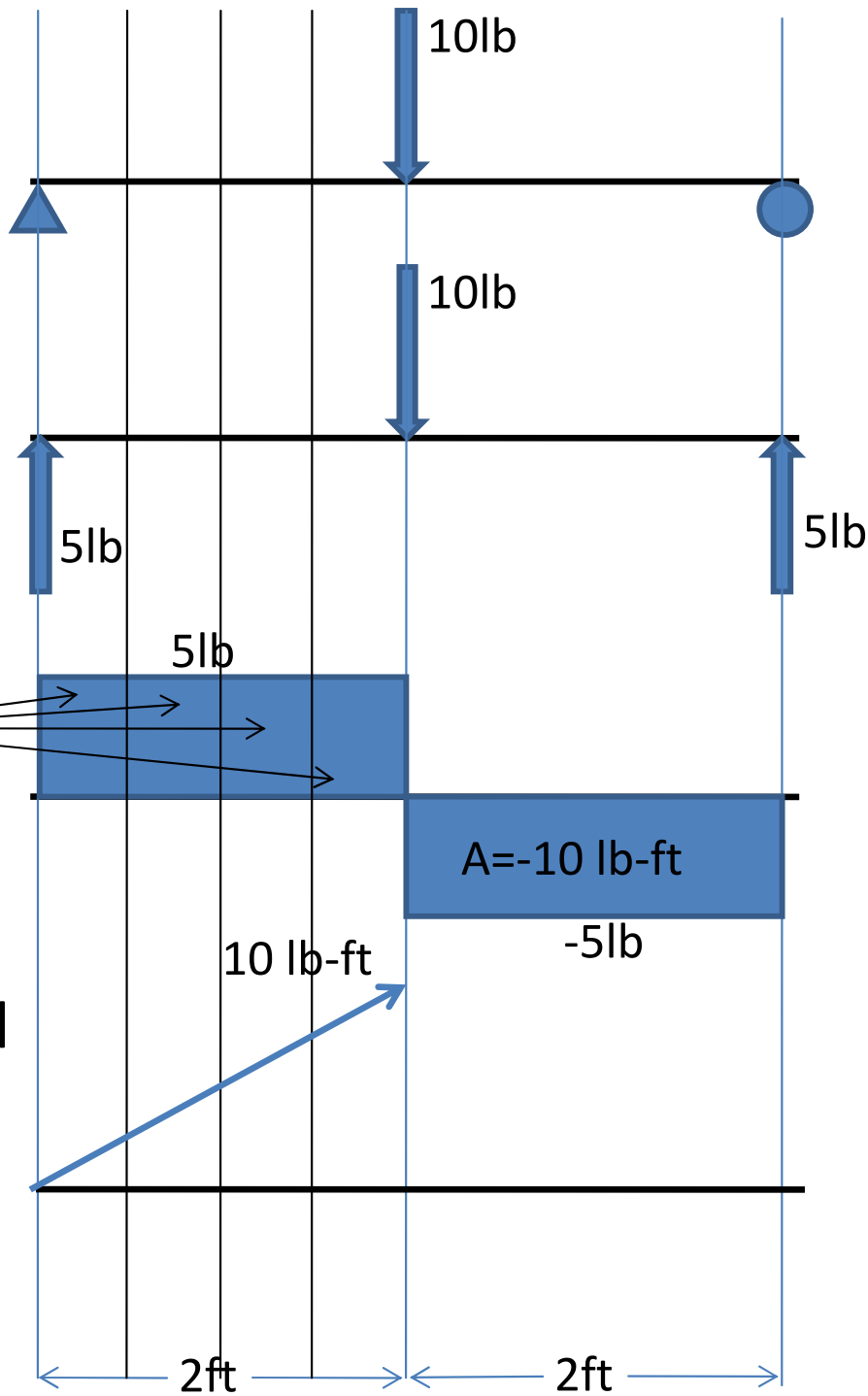
$A=-10 \text{ lb-ft}$

-5lb

7.5 lb-ft

2ft

2ft



FB

D

V

M

$A=2.5 \text{ lb-ft}$

AREA under V diagram is equal to the CHANGE in the M diagram

$A=-10 \text{ lb-ft}$

10 lb-ft

-5 lb

5 lb

5 lb

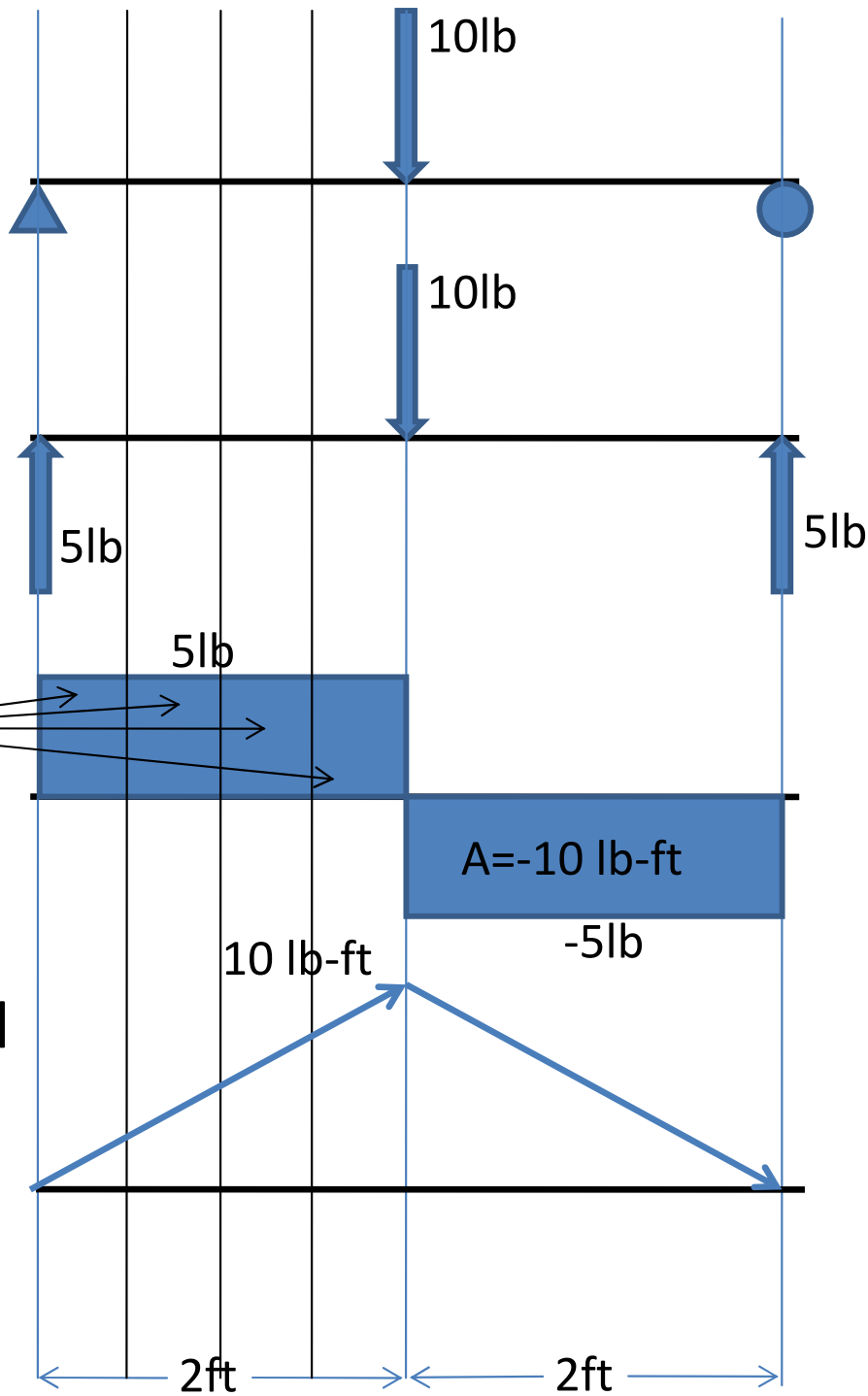
10 lb

10 lb

5 lb

2 ft

2 ft



FB

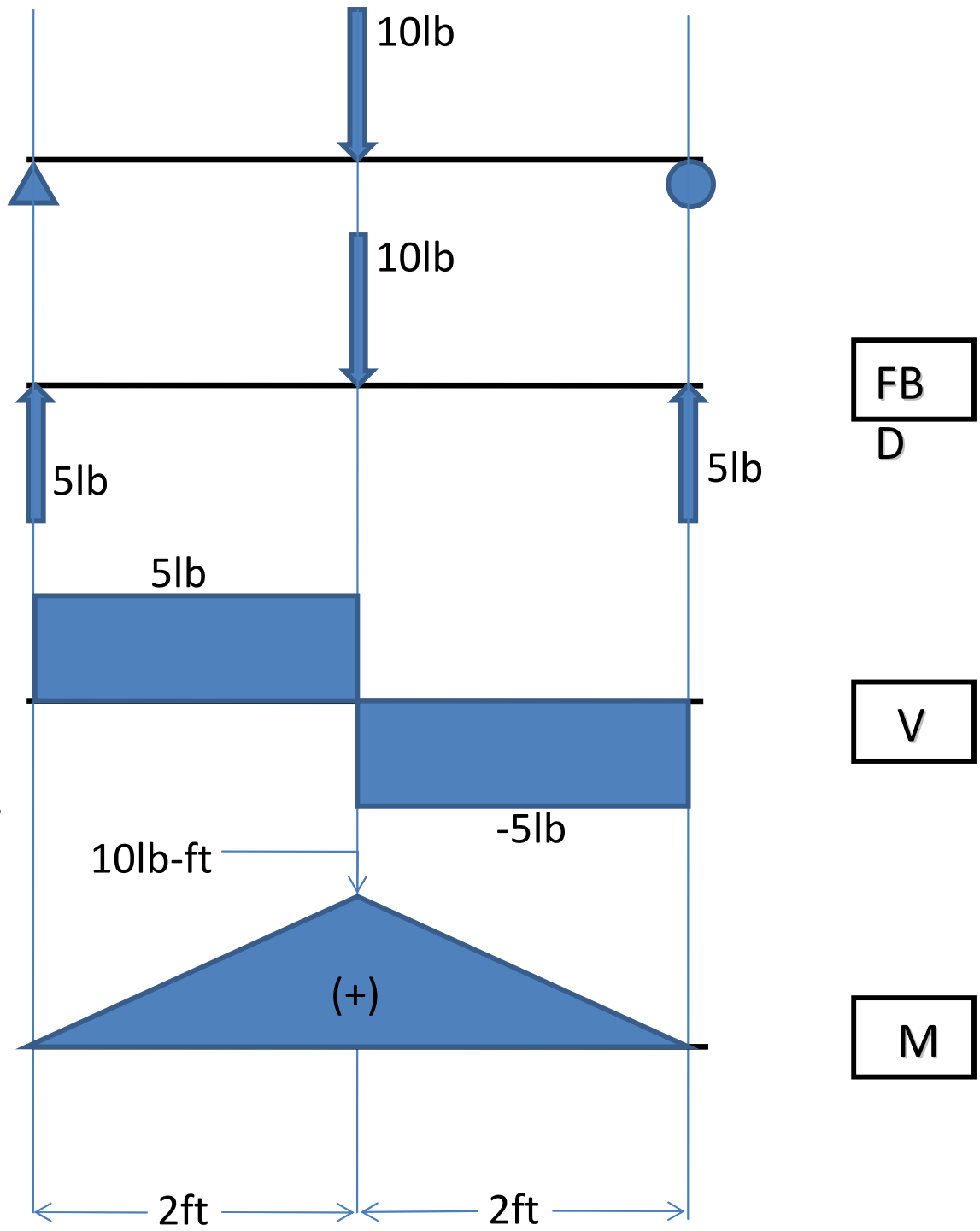
D

V

M

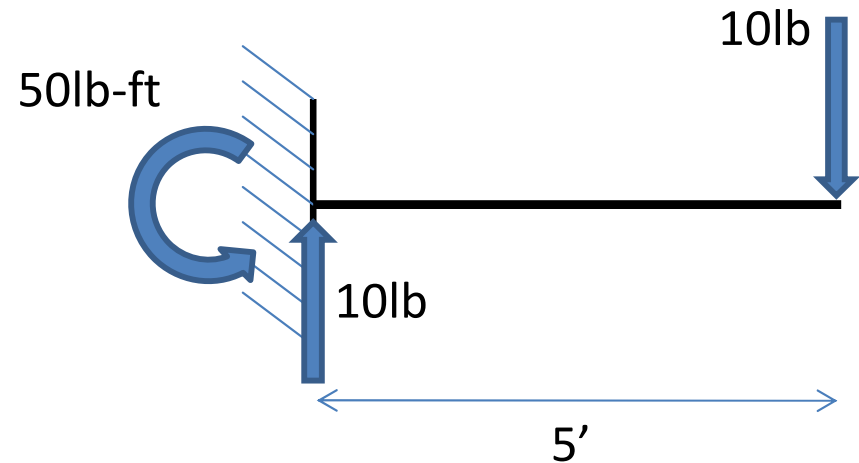
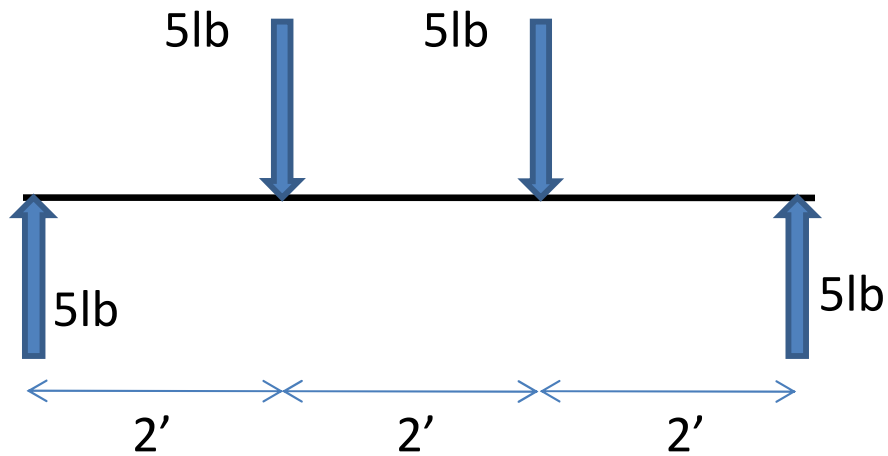
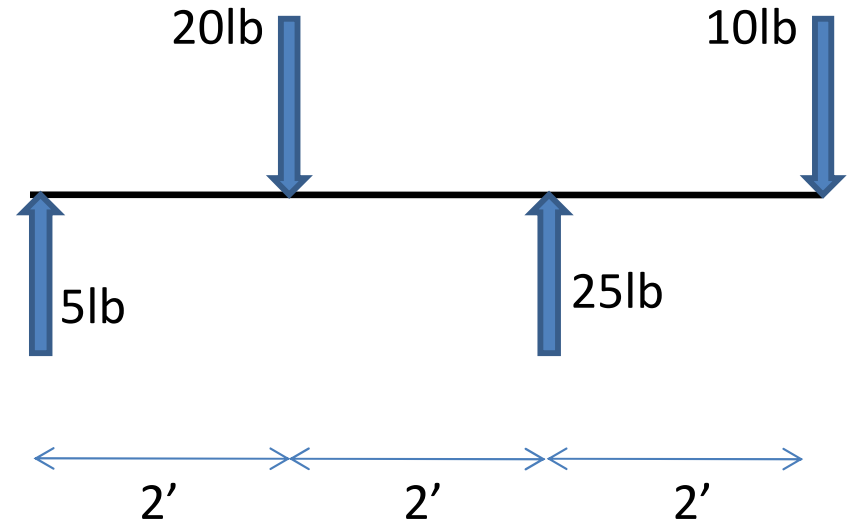
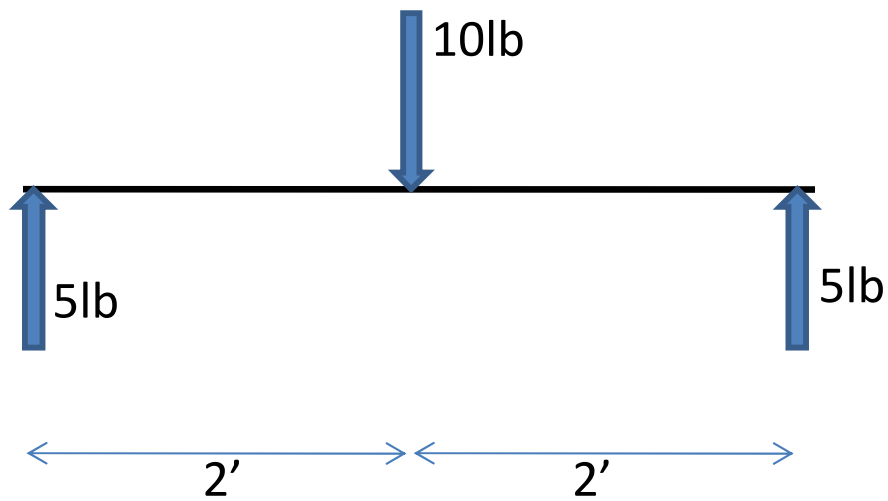
$A = 2.5 \text{ lb-ft}$

AREA under V diagram is equal to the CHANGE in the M diagram



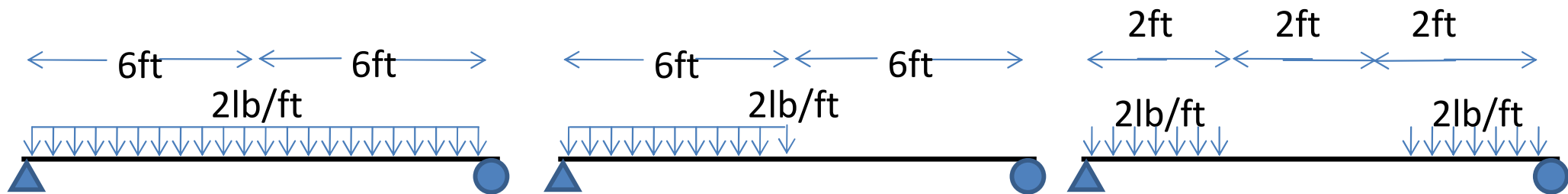
Notice: Value of V diagram is slope of M diagram

In-Class Examples

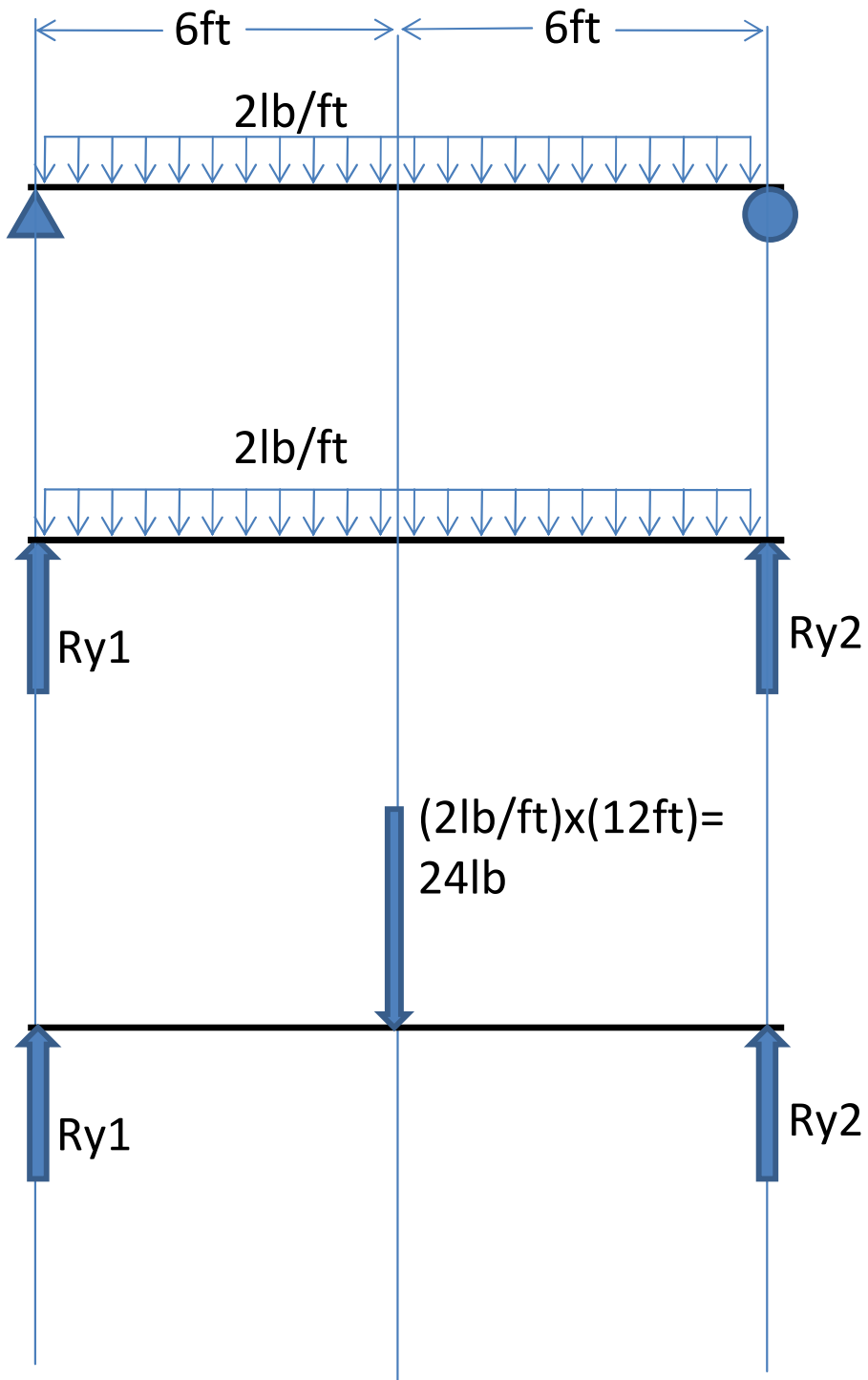


Moment-Shear Diagrams for Distributed Loads

Finding Reactions for beams with a distributed load

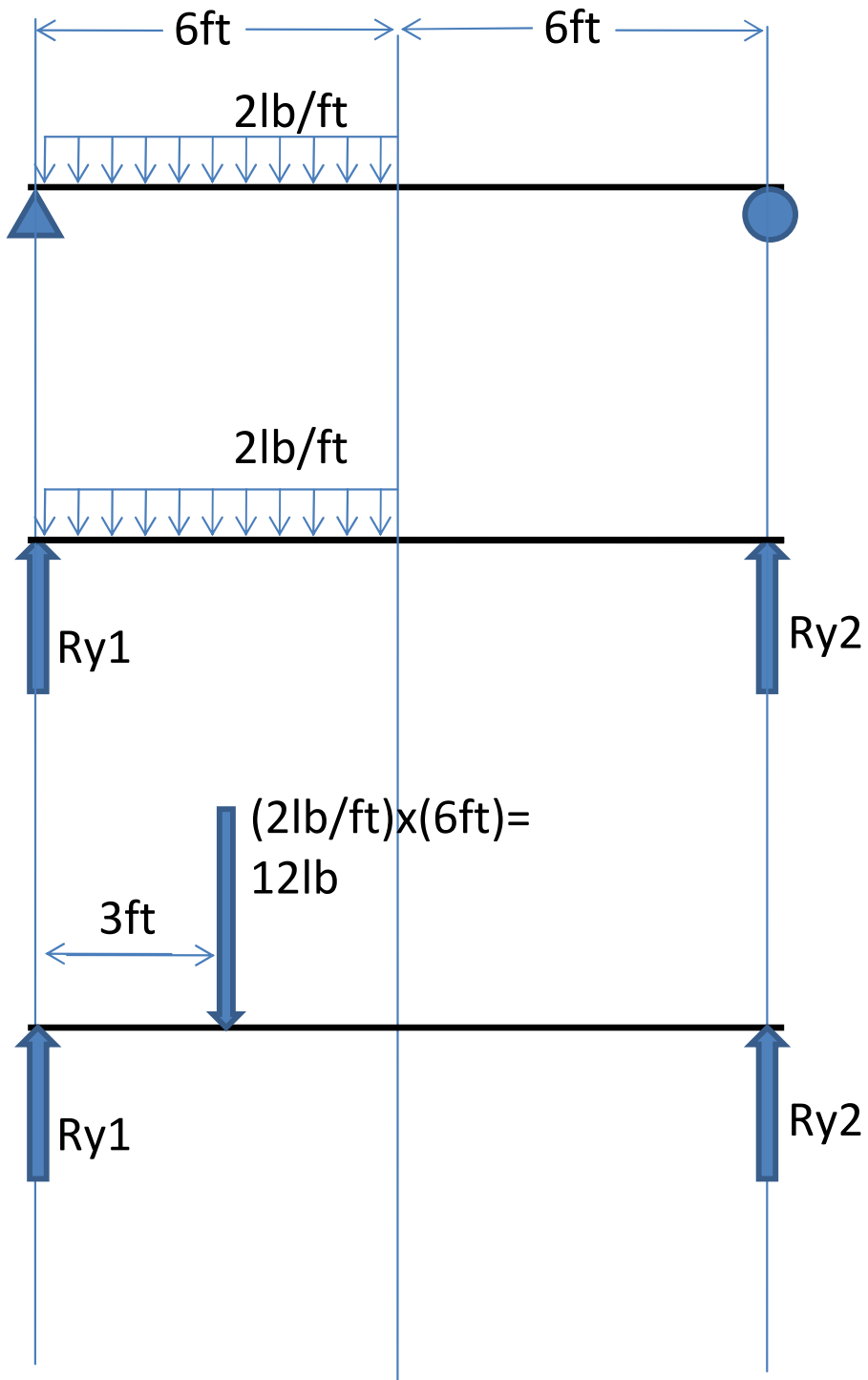


- For the purpose of FINDING THE REACTIONS for a beam with a distributed load, the distributed load can be assumed to be a concentrated load at the center of the distributed load.



$$\begin{aligned} \Sigma M_a &= 0 \\ -(24\text{lb})(6\text{ft}) + (R_{y_2})(12\text{ft}) &= 0 \\ R_{y_2} &= 12\text{lb} \end{aligned}$$

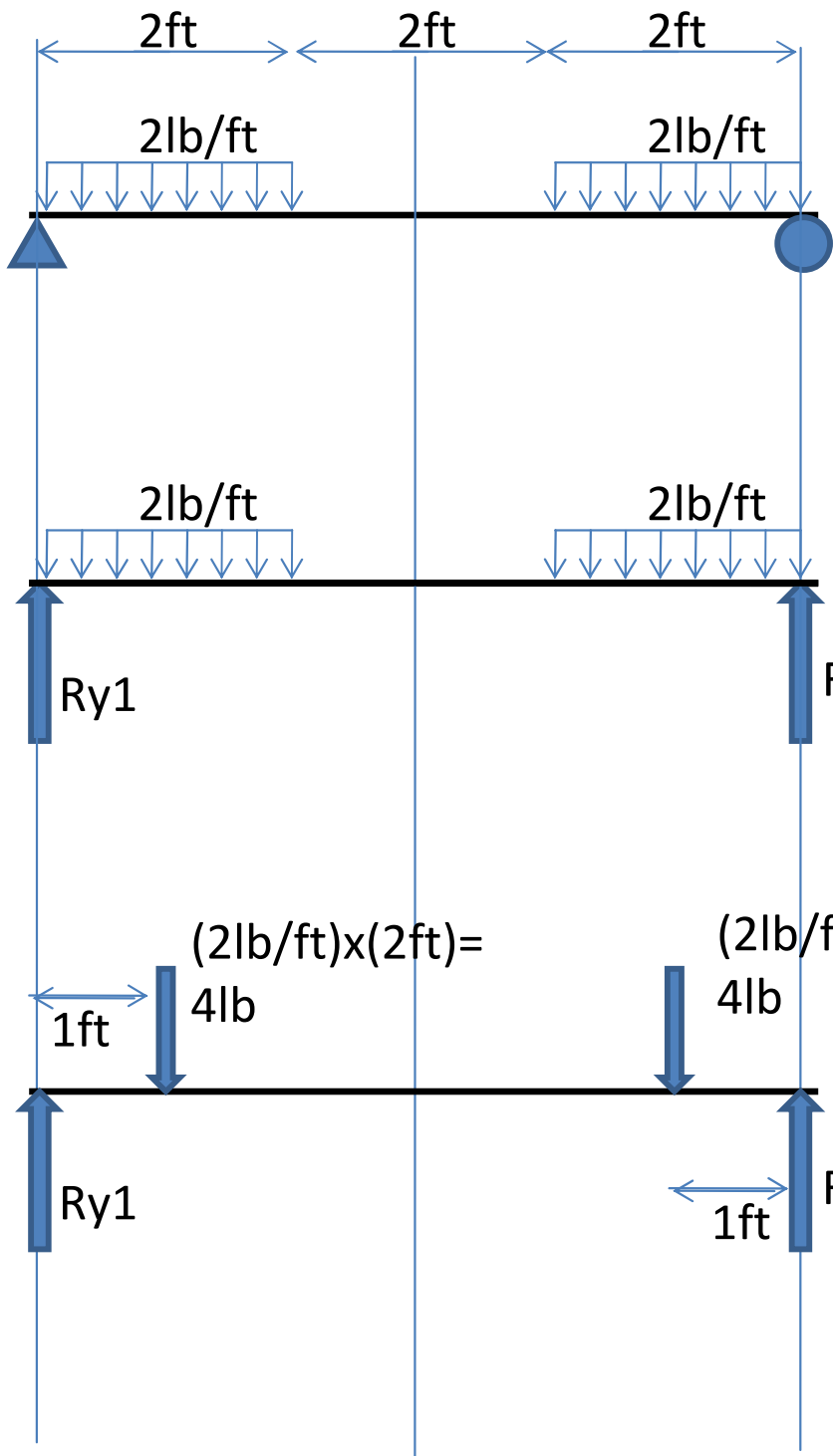
$$\begin{aligned} \Sigma F_y &= 0 \\ R_{y_1} - 24\text{lb} + 12\text{lb} &= 0 \\ R_{y_1} &= 12\text{lb} \end{aligned}$$



FB
D

$$\begin{aligned} \Sigma M_a &= 0 \\ -(12\text{lb})(3\text{ft}) + (R_{y_2})(12\text{ft}) &= 0 \\ R_{y_2} &= 3\text{lb} \end{aligned}$$

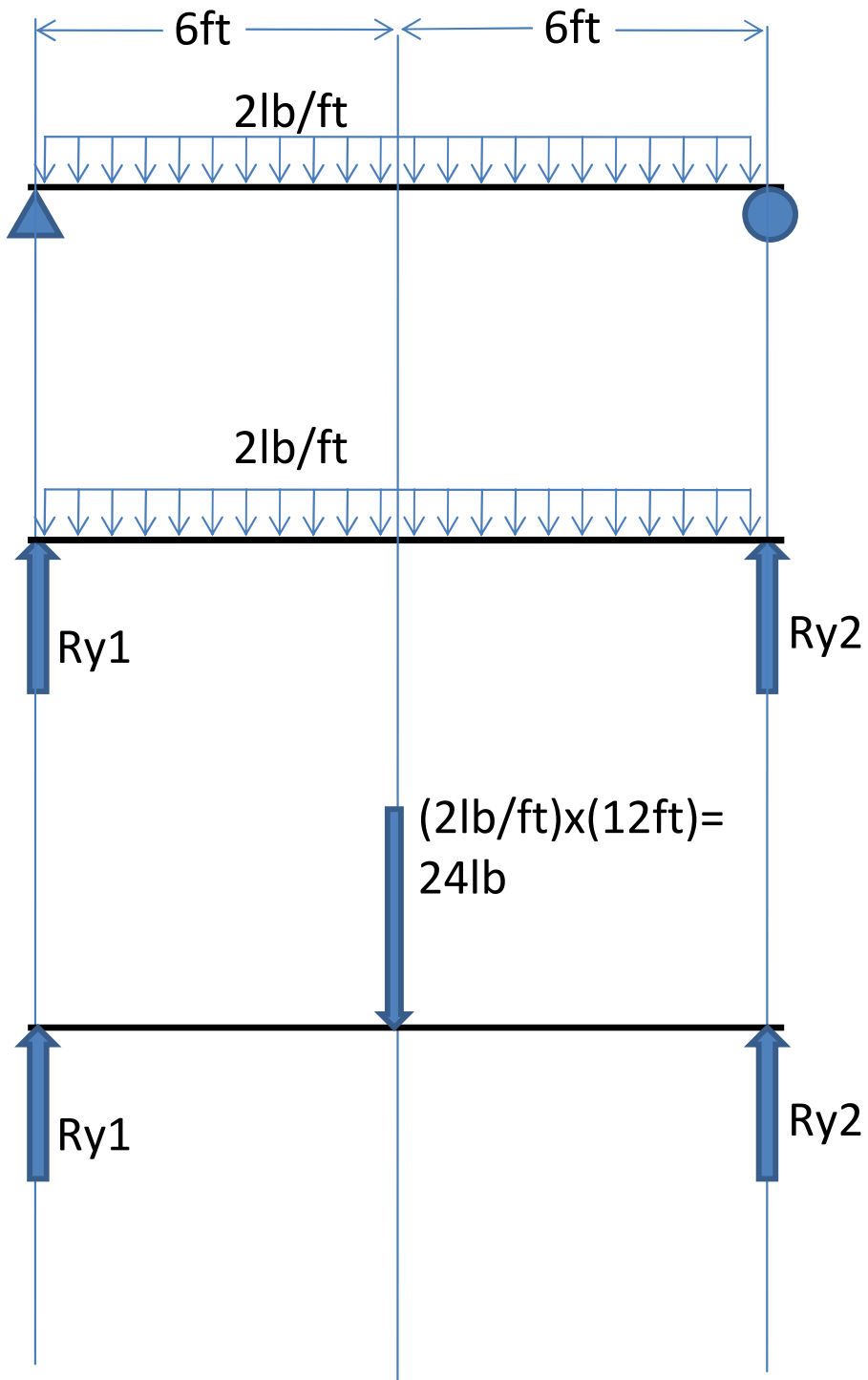
$$\begin{aligned} \Sigma F_y &= 0 \\ R_{y_1} - 12\text{lb} + 3\text{lb} &= 0 \\ R_{y_1} &= 9\text{lb} \end{aligned}$$



FB
D

$$\begin{aligned} \Sigma M_a &= 0 \\ -(4\text{lb})(1\text{ft}) - (4\text{lb})(5\text{ft}) + (R_{y_2})(6\text{ft}) &= 0 \\ R_{y_2} &= 4\text{lb} \end{aligned}$$

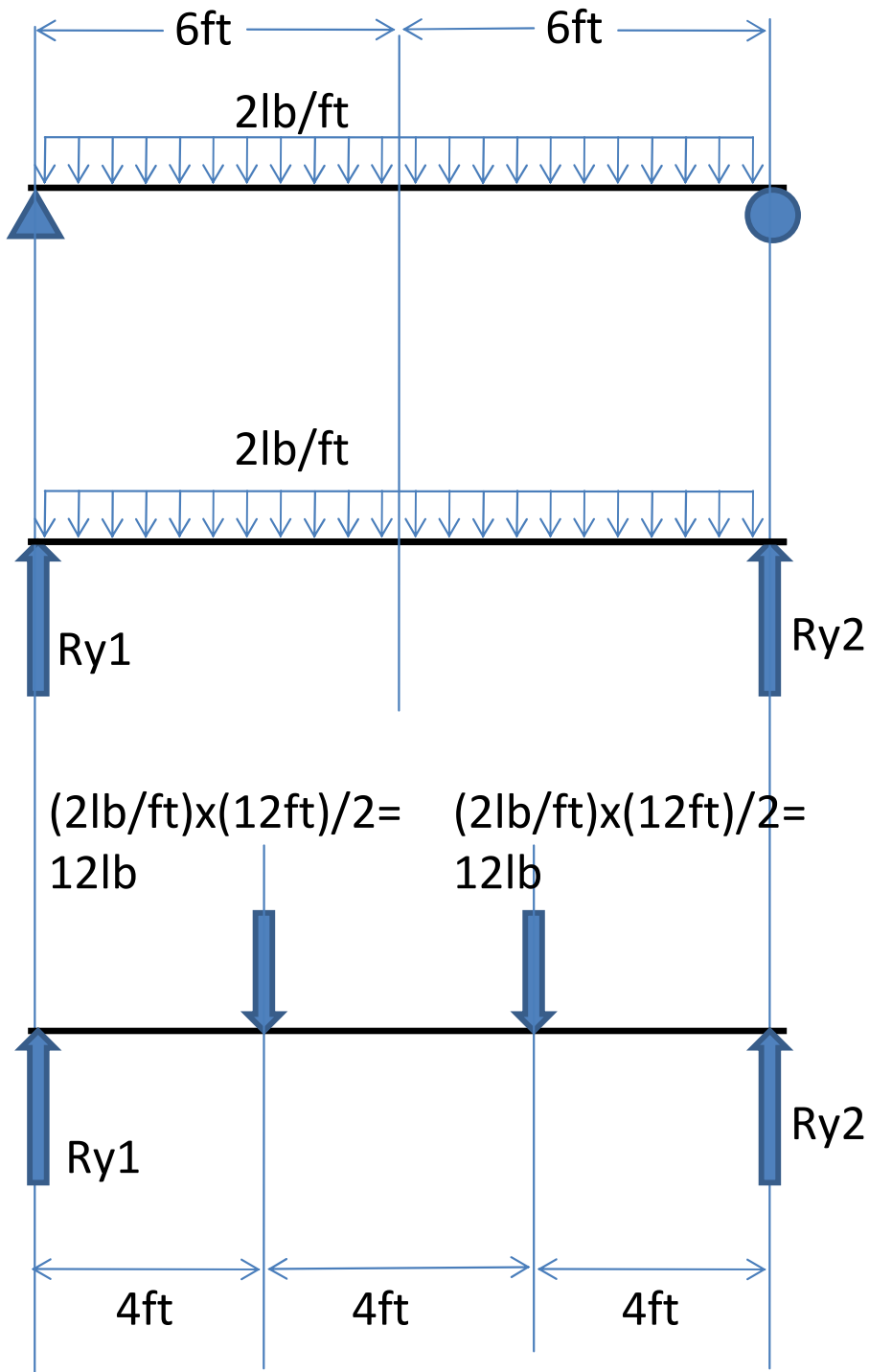
$$\begin{aligned} \Sigma F_y &= 0 \\ R_{y_1} - 4\text{lb} - 4\text{lb} + 4\text{lb} &= 0 \\ R_{y_1} &= 4\text{lb} \end{aligned}$$



FB
D

$$\begin{aligned} \Sigma M_a &= 0 \\ -(24\text{ lb})(6\text{ ft}) + (R_{y_2})(12\text{ ft}) &= 0 \\ R_{y_2} &= 12\text{ lb} \end{aligned}$$

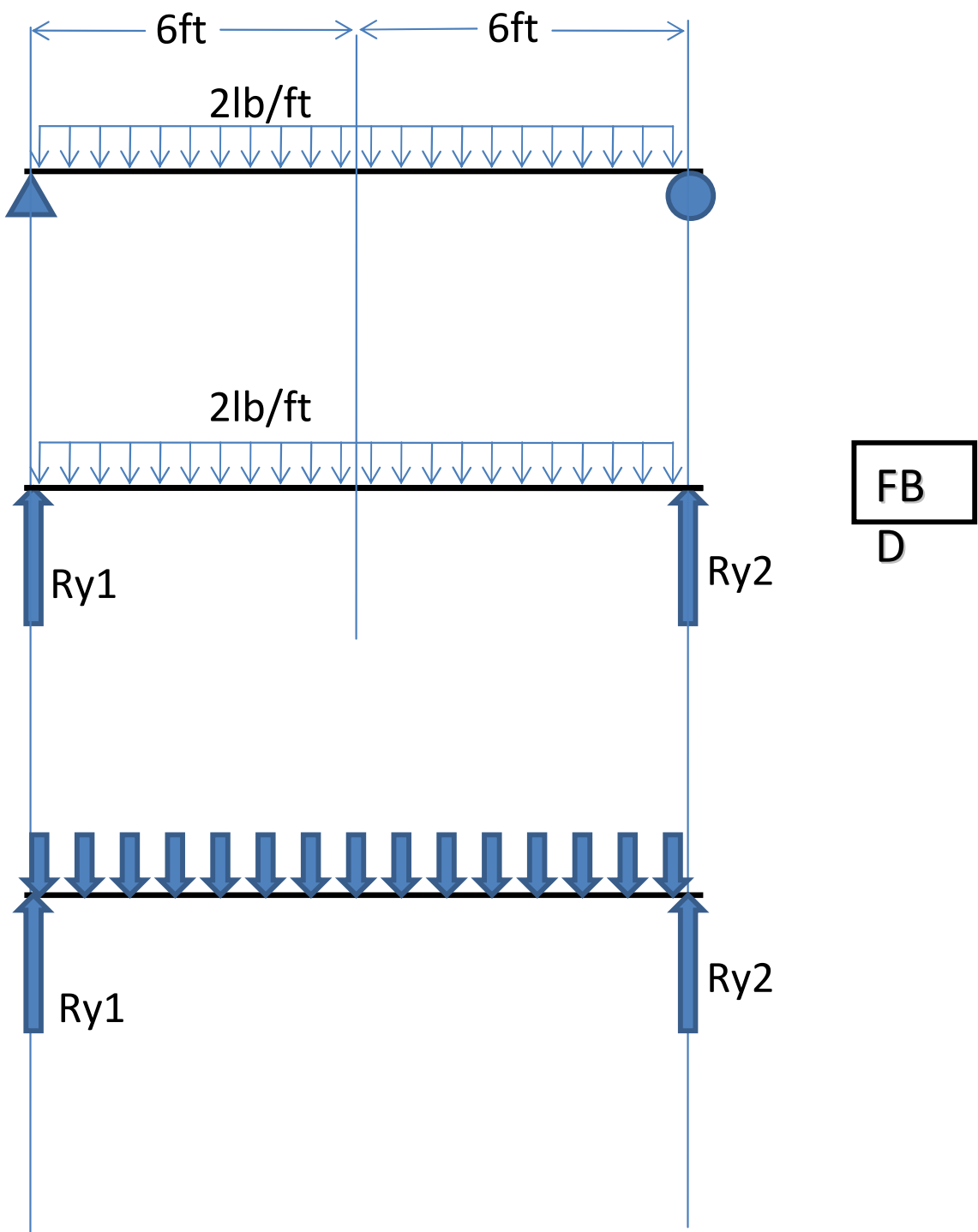
$$\begin{aligned} \Sigma F_y &= 0 \\ R_{y_1} - 24\text{ lb} + 12\text{ lb} &= 0 \\ R_{y_1} &= 12\text{ lb} \end{aligned}$$

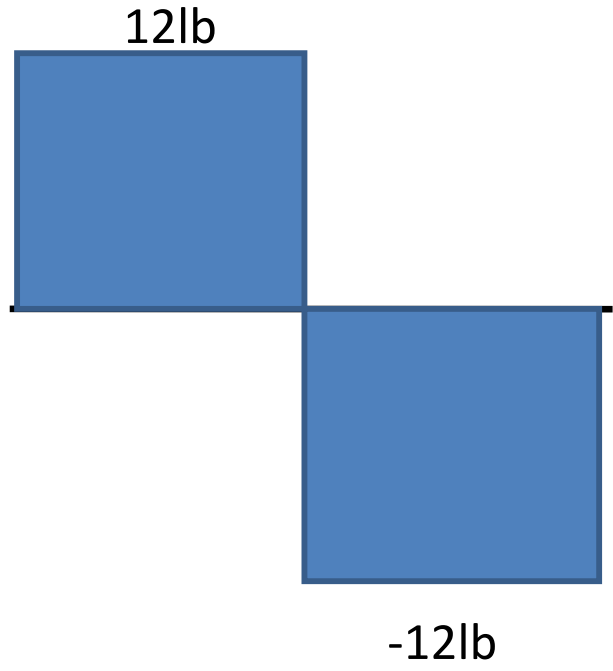
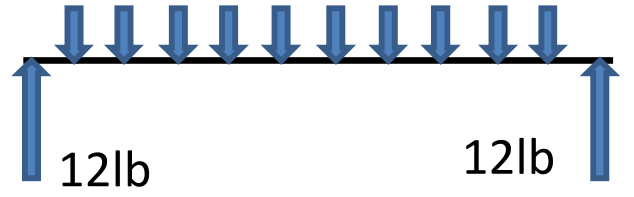
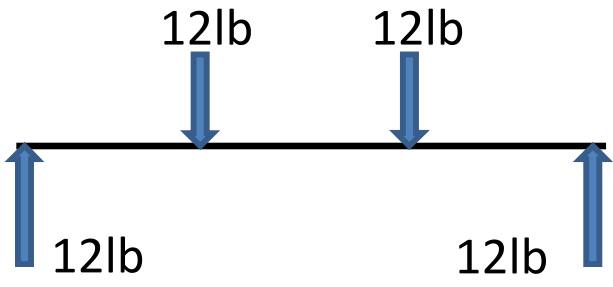
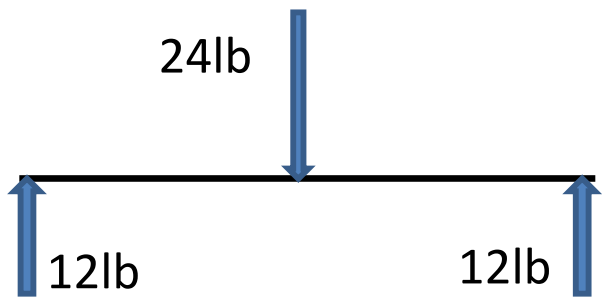


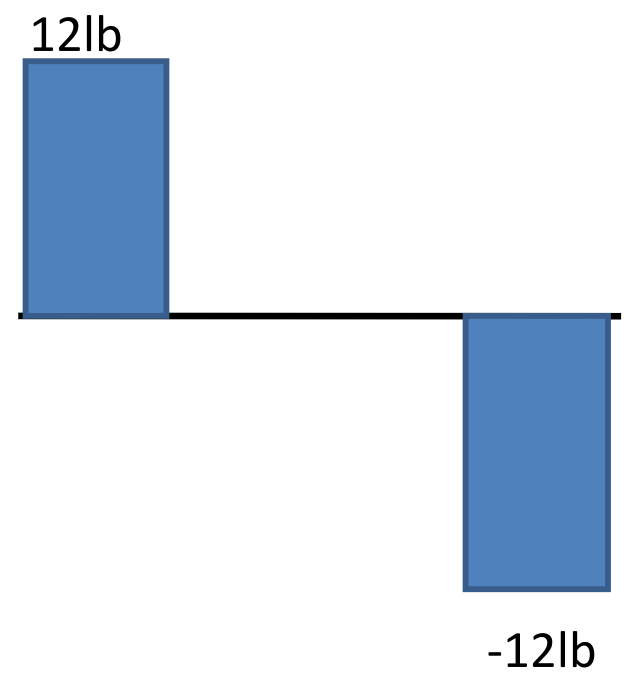
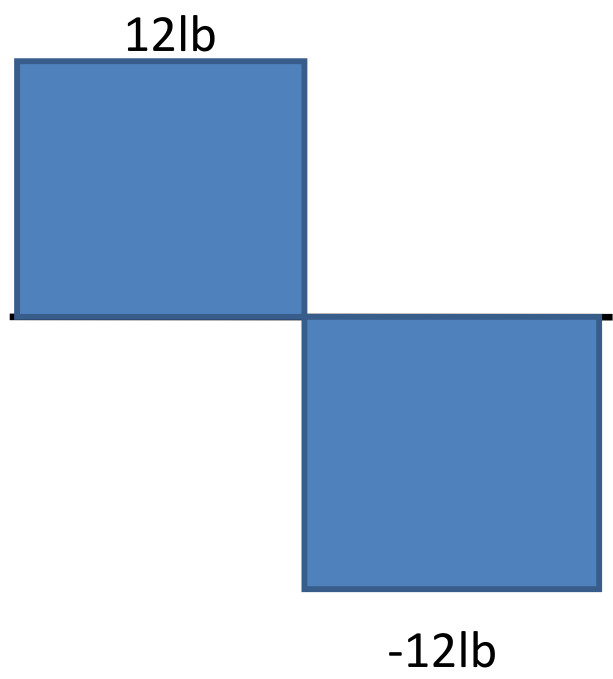
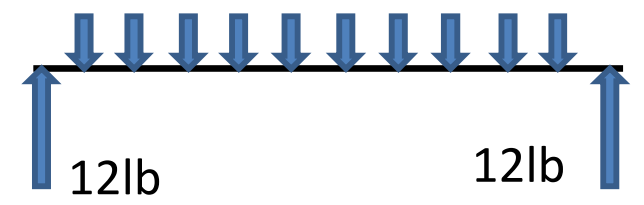
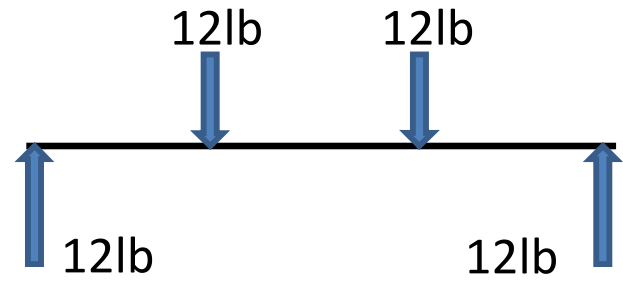
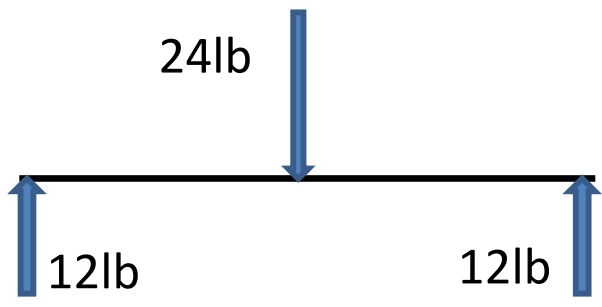
FB
D

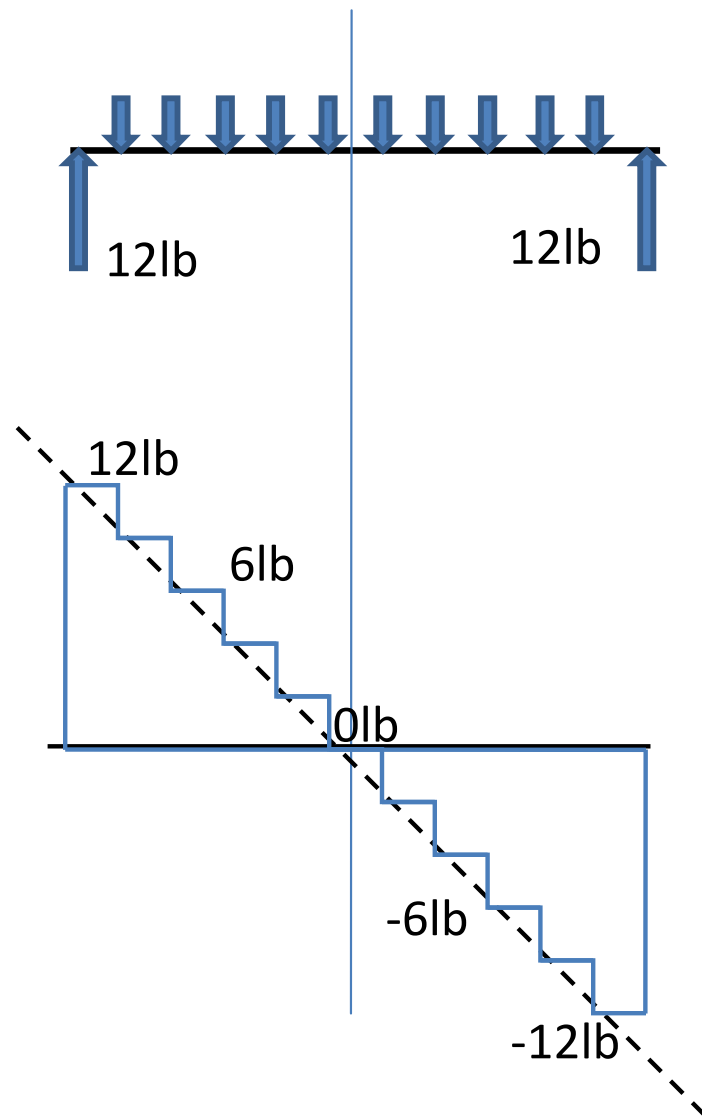
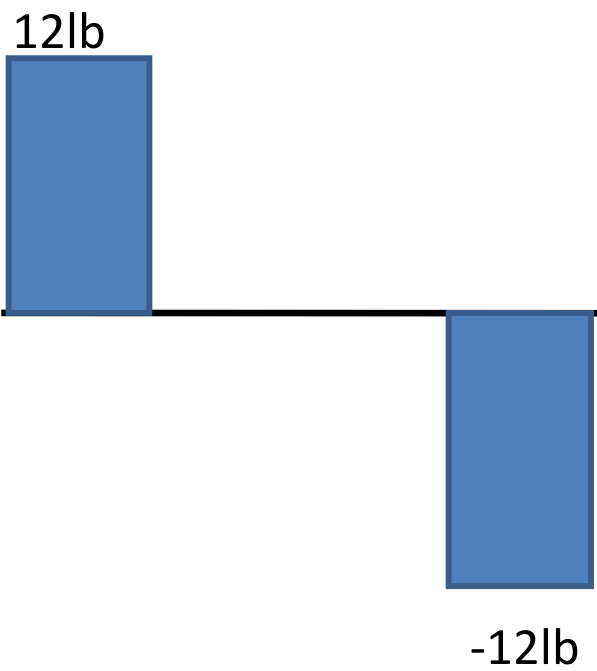
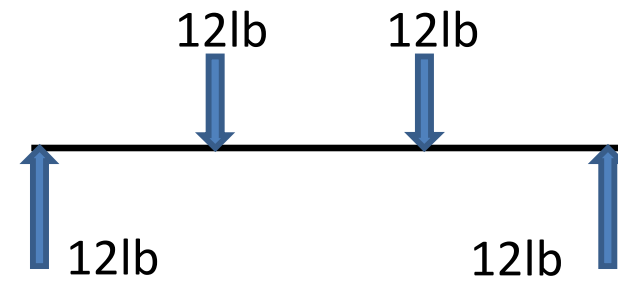
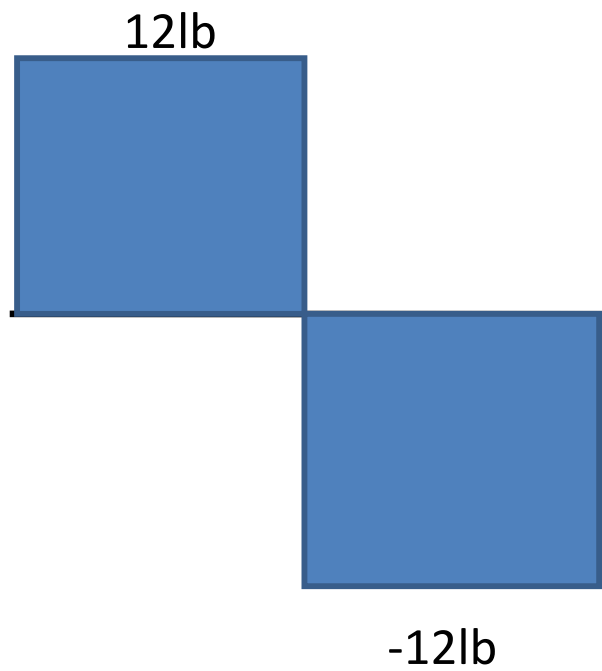
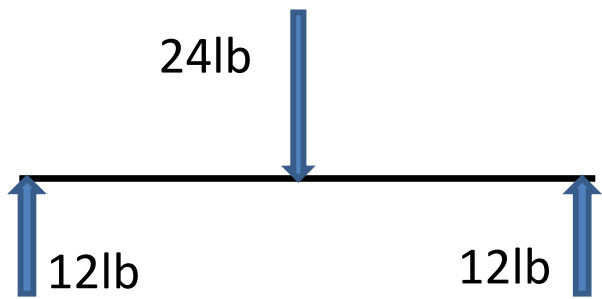
$$\begin{aligned} \Sigma M_a &= 0 \\ -(12\text{lb})(4\text{ft}) - (12\text{lb})(8\text{ft}) + (R_{y_2})(12\text{ft}) &= 0 \\ -48\text{lb-ft} - 96\text{lb-ft} &= -(R_{y_2})(12\text{ft}) \\ -144 / -12 &= R_{y_2} \\ R_{y_2} &= 12\text{lb} \end{aligned}$$

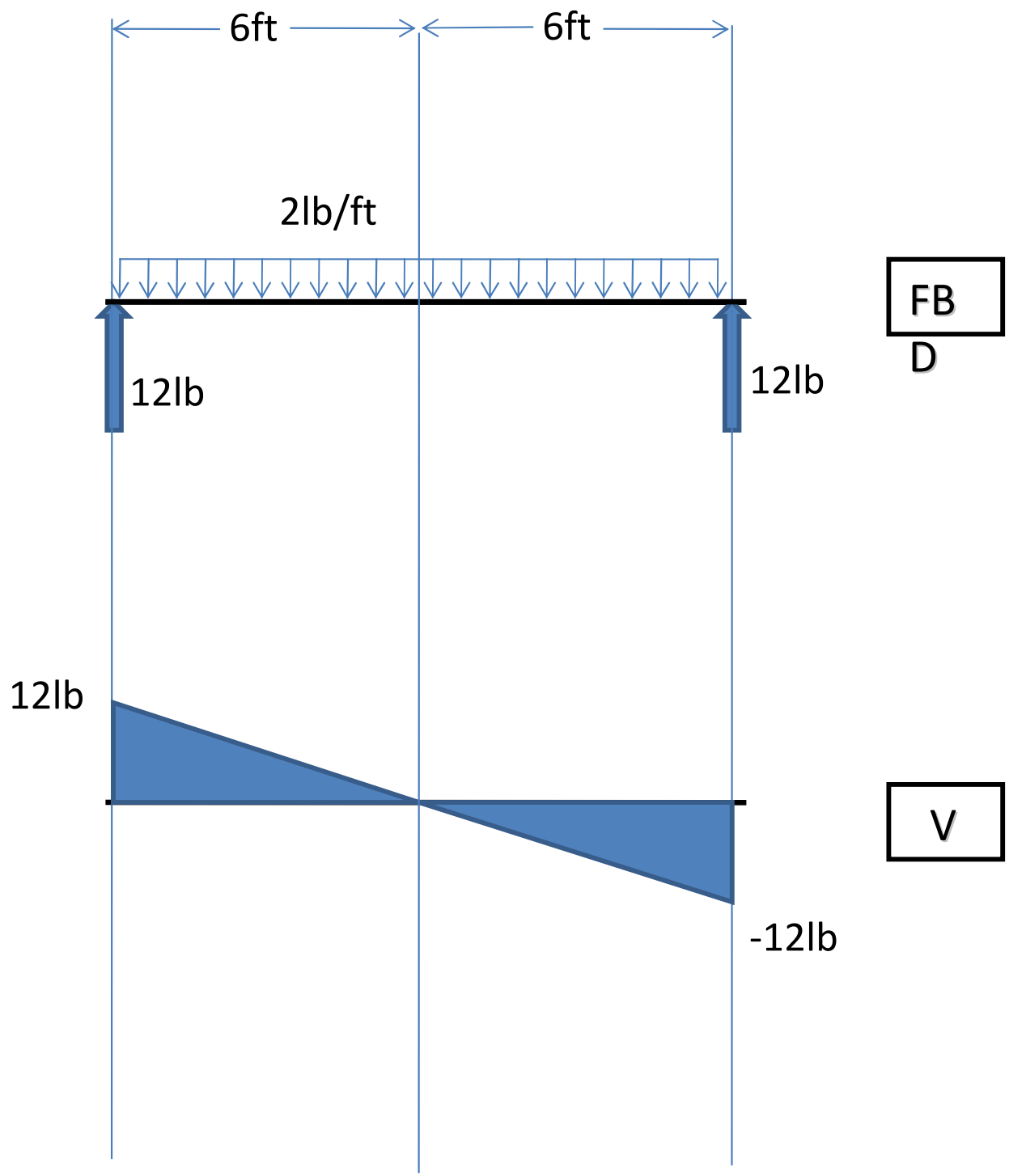
$$\begin{aligned} \Sigma F_y &= 0 \\ R_{y_1} - 12\text{lb} - 12\text{lb} + 12\text{lb} &= 0 \\ R_{y_1} &= 12\text{lb} \end{aligned}$$

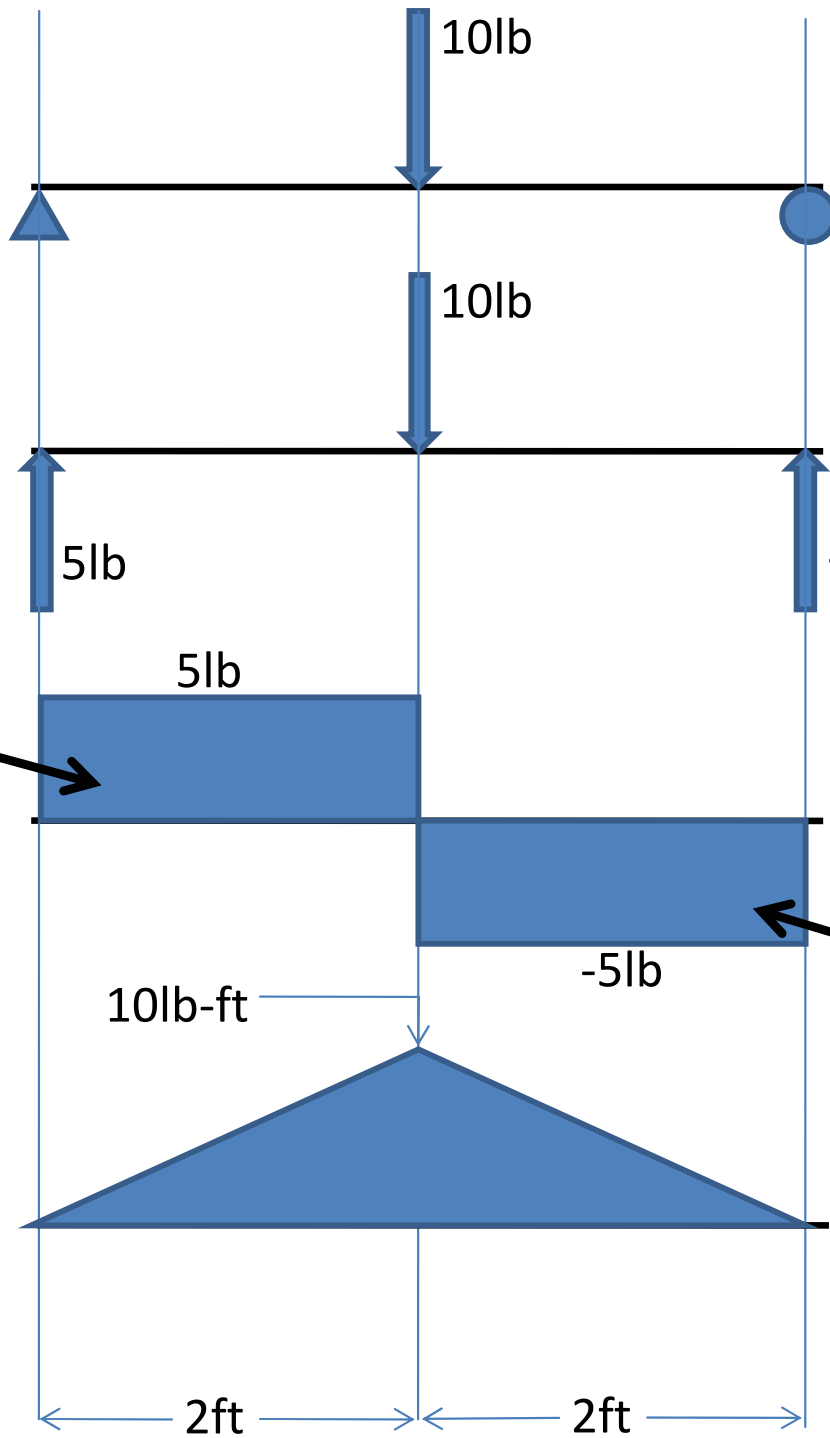












FB
D

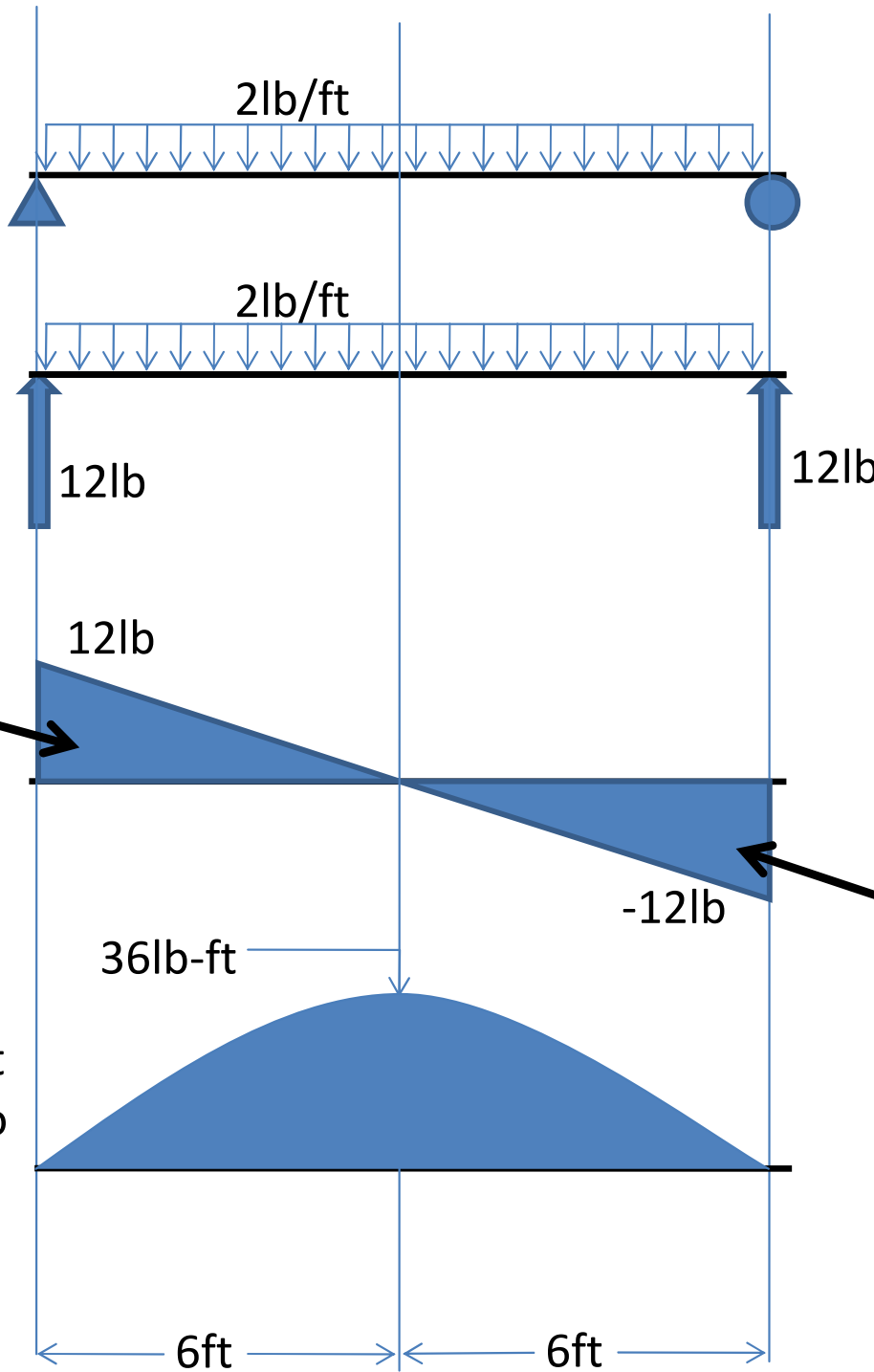
V

M

$A = (5lb)(2ft) = 10lb-ft$

$A = (-5lb)(2ft) = -10lb-ft$

Change in Moment Diagram is equal to the area under the shear diagram



FB

D

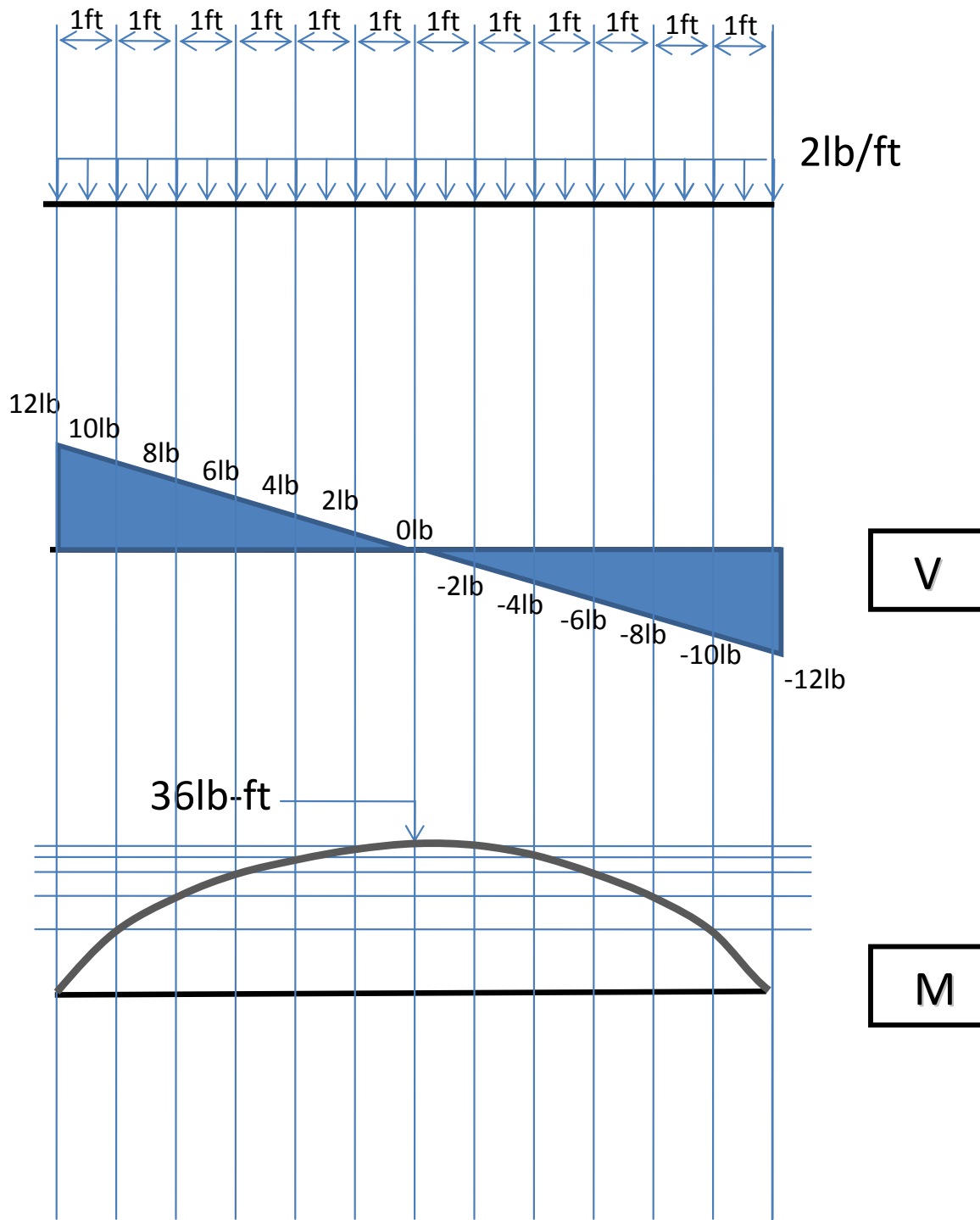
V

$$A = (-12\text{lb})(6\text{ft})/2 = -36\text{lb-ft}$$

M

$$A = (12\text{lb})(6\text{ft})/2 = 36\text{lb-ft}$$

Change in Moment Diagram is equal to the area under the shear diagram



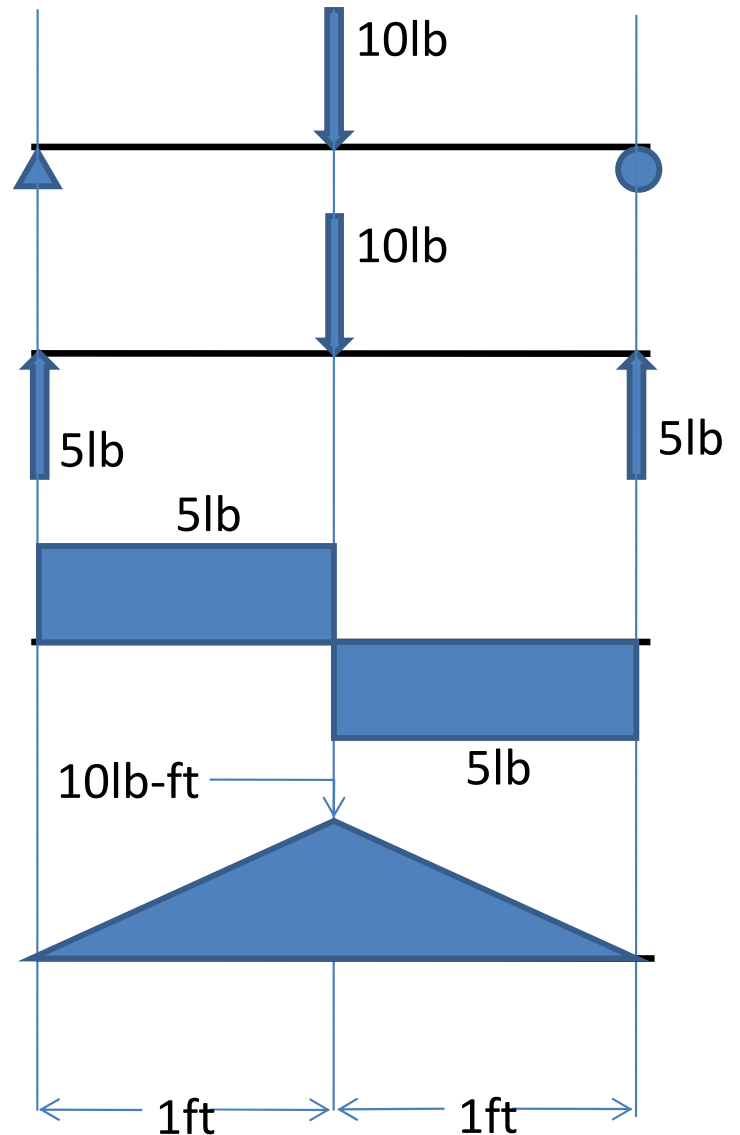
The VALUE at a point in the shear diagram is equal to the SLOPE of the moment diagram at that same point.

An external point load shifts the shear diagram an amount equivalent to its magnitude in its same direction.

A distributed load shifts the shear diagram linearly with a SLOPE equal to the magnitude of the distributed load in its same direction.

Change in Moment Diagram is equal to the area under the shear diagram.

The VALUE at a point in the shear diagram is equal to the SLOPE of the moment diagram at that same point.



An external point load shifts the shear diagram an amount equivalent to its magnitude in its same direction.

A distributed load shifts the shear diagram linearly with a SLOPE equal to the magnitude of the distributed load in its same direction.

Change in Moment Diagram is equal to the area under the shear diagram.

The VALUE at a point in the shear diagram is equal to the SLOPE of the moment diagram at that same point.

